

User's Guide

# PMOD Kinetic Modeling (PKIN)

Version 3.6



PMOD Technologies

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# PMOD Kinetic Modeling Tool (PKIN)

The PMOD kinetic modeling tool represents a flexible environment for the simulation and fitting of models over time. While initially aimed at the compartment models employed in Positron Emission Tomography (PET) and Single Photon Emission Computed Tomography (SPECT), models for other modalities such as magnetic resonance or optical imaging can also be easily incorporated due to the general approach and the plug-in structure for the models.

PKIN includes the following features:

- » import of time-vectors from measurements of blood and tissue activity via text files or directly from the PMOD Volume-Of-Interest (VOI) tool,
- » fitting of plasma fraction curves to derive tracer activity in plasma from whole blood activity,
- » fitting of metabolite correction curves to derive free tracer activity from total plasma activity,
- » representation of the plasma and whole-blood activity by a model for data smoothing and extension beyond the last measurement,
- » selection of a comprehensive list of compartment models, reference models and graphical plots,
- » weighted and non-weighted fitting of the selected model to the measurement whereby the parameter can be enabled or fixed, and can optionally be restricted within a physiological range,
- » coupled fitting of measurements from different tissues to improve the accuracy for parameters assumed to be identical across tissues,
- » calculation of goodness-of-fit criteria usable for model comparisons,
- » Monte Carlo simulations to assess the identifiability of model parameters using a standard or user-defined distribution of the measurement errors,
- » sensitivity analysis of compartment models to quantify the correlation between the model parameters,
- » visualization of the relative contributions of the different compartments to the model curve,
- » generation of synthetic studies representing compartmental kinetics for testing the performance of simplified pixel-wise models,
- » batch mode operation to fit or Monte Carlo simulate a set of prepared studies,
- » saving of the fitted parameters into text files and their aggregation for further statistical investigations.

Please note that the following description is intended as a reference and not as a tutorial. For practical examples how to work with the PKIN software please refer to the **PMOD Workbook**, which is distributed as part of the PMOD documentation.

# Introduction to Modeling in PET

For the data processing in PKIN it will be helpful to understand the basic modeling concepts in PET. The following sections present short introductions of compartment and reference models. Other model types are described in the *PKIN Model Reference* (on page 161) section.

For in-depth understanding of PET Kinetic Modeling we strongly recommend the PMOD users to attend one of the excellent yearly *PET Pharmacokinetics Courses*. These courses include theory as well as practical work and are organized by the the top experts in the domain. The 2015 course will be scheduled prior to the BrainPET Meeting in Vancouver (June 27-30, 2015). Two other worthwhile courses are *PET Methodology* (King's College London, March 2-4, 2015) and *Basic Kinetic Modeling in Molecular Imaging* (Copenhagen, March 2-6, 2015).

## Compartment Models

A physiologic system is often described by decomposition into a number of interacting subsystems, called *compartments*. Compartments should not be understood as a physical volume, but rather as a mass of well-mixed, homogeneous material that behaves uniformly. Each compartment may exchange material with other compartments. Examples of frequently used compartments are

- ▶▶ Authentic (unchanged) tracer in the arterial plasma that can be extracted into the tissue. The concentration of authentic tracer as a function of time during the PET acquisition is called the "Arterial Input Function" (AIF), or simply the "Input Curve".
- ▶▶ Free tracer in tissue that can be bound or that may diffuse back into the blood.
- ▶▶ Tracer in tissue that has been specifically bound, for instance, at the target receptor site.
- ▶▶ Tracer in tissue that has been non-specifically bound to other than the targeted cell components.

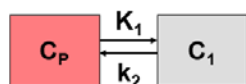
Compartment models are visualized by diagrams wherein the rectangles symbolize compartments and the arrows represent material exchange, as illustrated for the different models below. Although the mechanisms of material transport between compartments may differ, models that can be reasonably analyzed with standard mathematical methods assume first-order processes. As a consequence, the change of tracer concentration in each compartment is a linear function of the concentrations in the other compartments. Because of the tiny amounts of tracer material applied in nuclear medicine it is usually assumed that the observed system is not disturbed by the tracer. It is furthermore assumed, that the physiologic conditions do not change during the study, so that the rate of the material exchange can be considered constant.

## Model Structure

In the following, the compartment models are illustrated using a tracer that is injected intravenously as a bolus and is not freely diffusible. Please refer to the consensus paper of *Innis et al* <http://www.nature.com/jcbfm/journal/v27/n9/full/9600493a.html> [56] for the details of the nomenclature and the interpretation of the models in the context of brain receptor tracers.

When the tracer arrives in the heart chambers, it is well mixed with blood and distributed by the arterial circulation. It finally arrives at the capillary bed where exchange with the tissue can take place. Some fraction of the tracer is extracted into tissue and metabolized, the rest is transported back to the heart, from where a new circulation starts.

This simplified "physiologic" model can be translated into a 1-tissue compartment model (1TCM):



Because of mixing it is assumed that, the arterial tracer concentration is the same throughout the body, so that it can be measured in a peripheral artery. The tracer concentration in tissue



$C_1(t)$  increases by the extraction of tracer from the arterial blood plasma. A usual condition for a quantitative analysis is, that only the unchanged tracer, called the *authentic tracer* or *parent*, can enter tissue, whereas labeled metabolites which may also circulate cannot. Because extraction is described by a first-order process, the transfer of material is proportional to the parent concentration  $C_p(t)$ . Parallel to the uptake, tracer in tissue is reduced by a backwards transfer (or washout), which is proportional to the concentration in tissue. Both processes compete, so that the change over time of the net tracer concentration in tissue ( $dC_1(t)/dt$ ) can be expressed by the following differential equation

$$\frac{dC_1(t)}{dt} = K_1 C_p(t) - k_2 C_1(t)$$

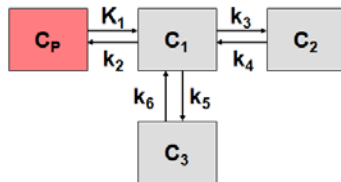
The two transfer coefficients,  $K_1$  and  $k_2$ , have a somewhat different meaning, indicated by using capital and small letters.  $K_1$  includes a perfusion-dependent component and has units of milliliter per minute per milliliter tissue, whereas  $k_2$  [ $\text{min}^{-1}$ ] indicates the fraction of mass transferred per unit time. For example, if  $k_2$  equals  $0.1[\text{min}^{-1}]$ , material leaves the tissue compartment at a rate of 10% per minute. The parent concentration  $C_p(t)$  is not a compartment, but considered as a measured quantity and appears as the input curve driving the system.

The differential equation of the 1-tissue compartment model can be analytically solved (integrated), yielding the following equation for the concentration of tracer in tissue:

$$C_1(t) = K_1 C_p(t) \otimes e^{-k_2 t} = K_1 \int_0^t C_p(\tau) e^{-k_2(t-\tau)} d\tau$$

Hence the time course of the tissue concentration essentially results from a convolution  $\otimes$  of the input curve with a decaying exponential.  $K_1$  acts as a scaling factor of the concentration course, whereas the washout coefficient  $k_2$  has an impact on its form.

More complex compartment models distinguish different forms of tracer in tissue, which is usually interpreted as follows. After entering a cell, the tracer is available for binding in a free form at a concentration  $C_1(t)$ . Free tracer can directly be bound to its target molecule,  $C_2(t)$ , but it also may bind to some cell components that are not known in detail,  $C_3(t)$ . Considering these pathways, a 3-tissue compartment model (**3TCM**) with six transfer coefficients is established, whereby  $C_2$  and  $C_3$  communicate only by  $C_1$ .



The system of differential equations can be derived in analogy to the 1-tissue compartment model, but is much more complex. The equations describing this model are given by

$$\begin{aligned}\frac{dC_1(t)}{dt} &= K_1 C_p(t) - (k_2 + k_3 + k_5) C_1(t) + k_4 C_2(t) + k_6 C_3(t) \\ \frac{dC_2(t)}{dt} &= k_3 C_1(t) - k_4 C_2(t) \\ \frac{dC_3(t)}{dt} &= k_5 C_1(t) - k_6 C_3(t)\end{aligned}$$

This system contains six unknown parameters ( $K_1$  to  $k_6$ ) that are difficult to assess experimentally. For practical purposes, the system is therefore most often reduced to a 2-tissue compartment model (**2TCM**) by treating free and non-specifically bound tracer as a single compartment  $C_1$  (non-displaceable compartment).



This simplification is justified if the exchange free  $\leftrightarrow$  nonspecific binding is significantly faster than the exchange free  $\leftrightarrow$  specific binding. Otherwise specific and nonspecific binding cannot be distinguished, and the resulting transfer coefficients cannot be attributed to specific binding alone. The simplified two-tissue compartment model is given by

$$\begin{aligned}\frac{dC_1(t)}{dt} &= K_1 C_p(t) - (k_2 + k_3) C_1(t) + k_4 C_2(t) \\ \frac{dC_2(t)}{dt} &= k_3 C_1(t) - k_4 C_2(t)\end{aligned}$$

By using a mathematical method called Laplace transformation, analytic solutions can be derived for linear multi-compartment models, but the mathematical formulas are complicated expressions already for two compartments. However, they demonstrate that generally  $K_1$  represents a scaling factor like in the solution given for the one-tissue compartment.

## Model Simplification

While complex models are a more realistic description of the involved processes, they include many parameters. PET data from dynamic measurements has a limited statistical quality, and represents the summed contributions from all compartments. When trying to estimate a large number of model parameters from such data, the variance of the resulting parameters tends to be too high for reliable interpretation. Therefore, only relatively simple models with 2 to 4 (at most 6) parameters are feasible in practice.

When the 3-tissue compartment model is simplified to a 2-tissue compartment model, and further to a 1-tissue compartment model, the meaning of the transfer coefficients changes (except for  $K_1$ ). The “lumped” parameters of the simplified models can be expressed by the parameters of the complex models as shown in the tables below. Note that  $k_2$  and  $k_3$  have a different meaning in the different models, as well as  $C_1$ .

Parameters in 1TCM	Expressed by parameters of the 2TCM	Expressed by parameters of the 3TCM

$K_1$	$K_1$	$K_1$
$k_2$	$\frac{k_2}{1 + k_3 / k_4}$	$\frac{k_2}{1 + k_3 / k_4 + k_5 / k_6}$

Parameters in 2TCM	Expressed by parameters of the 3TCM
$K_1$	$K_1$
$k_2$	$\frac{k_2}{1 + k_5 / k_6}$
$k_3$	$\frac{k_3}{1 + k_5 / k_6}$
$k_4$	$k_4$

The term  $1/(1+k_5/k_6)$  represents the free fraction of tracer in the non-displaceable tissue compartment which is available for transfer back to blood or for specific binding, and is often called  $f_2$ .

A good summary on the configuration and interpretation of linear models for receptor tracers is given by *Koepp et al.* <http://www.> [15]. Starting from the 3-tissue model it is nicely shown how the parameters of simplified models are related to the true rate constants. **PKIN** can take these relations into account when switching from a more complex model to a simpler one. This behavior can be switched on by the **Model Conversion** check on the **Extras** panel.

### Relation of K1 with Perfusion

As mentioned earlier,  $K_1$  is related to tissue perfusion  $F$ . With a capillary model, the relation

$$K_1 = E F$$

can be derived.  $E$  represents the unidirectional first-pass extraction fraction (i.e., the fraction of tracer that penetrates the capillary wall and is extracted into the tissue during the first pass of tracer through the capillary). The reverse transport is not regarded in the calculation of  $E$ . Renkin [40] and C. Crone [41] calculated the extraction as

$$E = 1 - e^{-PS/F}$$

$P$  denotes permeability, and  $S$ , the surface of the endothelium by which the capillary exchanges with the tissue. The relation demonstrates that the extraction depends on the properties of the endothelium with respect to the tracer ( $PS$ ), as well as the perfusion  $F$ . The higher the perfusion, the smaller the extraction, because the average time spent close to the capillary wall decreases. However, total mass transport in general still increases with flow, as the reduced extraction is more than compensated by the higher abundance of tracer ( $F$  in the term  $EF$ ). For tracers with very high permeability, the extraction is virtually independent

of perfusion and  $E$  approaches 1 (such as for  $^{15}\text{O}$ -labeled water). In this case,  $K_1$  equals perfusion.

### Volumes of Distribution for Receptor Tracers

For receptor tracers, it is important that the extraction from plasma to tissue is sufficient, such that the kinetics is not limited by the extraction step. Additionally, the relation of the transfer coefficients for specific and nonspecific binding determines whether these different types of binding can be distinguished.

Naturally, the main interest is in specific binding. Besides the individual rate constants  $k_3$  and  $k_4$ , which are often difficult to estimate precisely, the distribution volume of specific binding  $V_s$  is a measure which is often used to quantify specific binding.  $V_s$  is defined by the ratio of specific binding concentration to total parent at equilibrium (which may not occur during the experiment). However, it can also be calculated from the rate constants of the 3-tissue compartment model as follows.

$$V_s = K_1/k_2 \cdot k_3/k_4 \quad \text{3-Tissue Compartment Model}$$

Under the assumption of a rapid equilibration between the free and non-specific compartment the same expression with the rate constants from the 2-tissue compartment model is often used as an approximation of  $V_s$ .

If the assumption of rapid equilibration is not justified one must resort to considering the entire tracer concentration in tissue as an indicator of specific binding. The measure used in these cases is called the "total distribution volume"  $V_T$ . In general it is defined as the ratio of the tracer concentration in tissue to total parent in plasma at equilibrium. A  $V_T$  equal to 20 hence means that the tracer is being concentrated in tissue by 20:1.  $V_T$  can be easily computed from the rate constants of the compartments model.

$$V_T = K_1/k_2 \quad \text{1-Tissue Compartment Model}$$

$$V_T = K_1/k_2 (1 + k_3/k_4) \quad \text{2-Tissue Compartment Model}$$

$K_1/k_2$  can be interpreted as the distribution volume of the first tissue compartment  $C_1$ . In receptor tracer experiments, this compartment is usually called the non-displaceable compartment, and  $K_1/k_2$  the distribution volume  $V_{ND}$  of non-displaceable uptake,

$$V_{ND} = K_1/k_2 \quad \text{2-Tissue Compartment Model}$$

This naming indicates that the concentration of free and non-specifically bound tracer in tissue is not reduced when a non-labeled ligand is brought into the tissue, competing with binding to the same receptor as the tracer. The rationale for this assumption is that the potential for non-specific binding to various cell components is practically unlimited as compared to the number of specific binding sites.

### Rate Constants and Binding Potential for Receptor Tracers

$k_2$  represents the fractional rate constant for the washout of free tracer from tissue to plasma. It is a composite rate constant and is related to flow in the same way as  $K_1$ . Consequently it

is often reasonable to assume that the ratio  $K_1/k_2$  is insensitive to flow and regionally constant. This property can be exploited as a restriction in a model fit, hereby reducing the number of fitted parameters.

$k_3$  is regarded as the rate of association of the ligand with the specific binding sites. It is a function of the concentrations of the free radioligand, the available binding sites and intrinsic second order association rate constants. Consequently it is a pseudo first order rate constant incorporating the concentration of available binding sites.

$k_4$  is considered as the dissociation rate constant of the receptor-ligand complex; often it is assumed to be invariant, what is still a matter of debate.

The binding potential (BP) quantifies the equilibrium concentration of specific binding as a ratio to some other reference concentration. Particularly, the ratio at equilibrium of specifically bound radioligand to that of non-displaceable radioligand in tissue is referring to as  $BP_{ND}$  (unitless) and equals the ratio  $k_3/k_4$ .

## Input Curve

With rare exceptions only a fraction of the tracer in blood is exchangeable with tissue and thus needs to be determined from blood aliquots which are withdrawn from an artery during the whole study.

A whole blood sample comprises tracer in the red blood cells and different tracer categories in the plasma: free unchanged tracer (parent), tracer bound to plasma proteins, tracer chemically changed by metabolite processes (metabolites). Depending on the tracer ligand, exchange processes are continuously going on between the different tracer pools in blood.

## Blood Sample Analytics

The following blood analysis steps are required in quantitative PET studies to determine the concentration of unchanged tracer in plasma as the Arterial Input Function (AIF).

Separation of the plasma from the red blood cells by centrifugation.

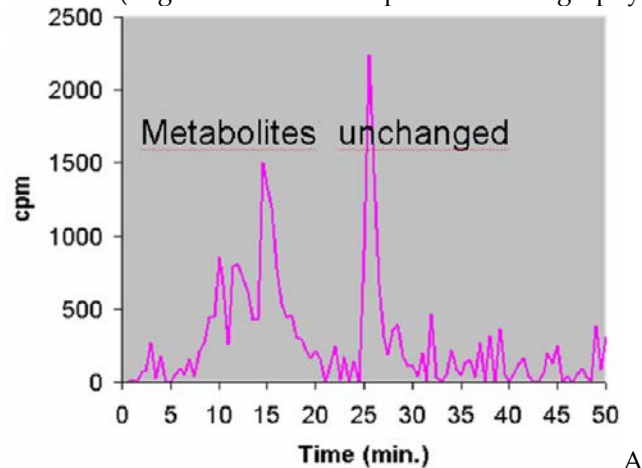


Plasma represents about 60% of the total blood volume.

Further separation of tracer in plasma into tracer metabolites and unchanged tracer by filtration



or HPLC (High Performance Liquid Chromatography).



- Determination of the free fraction of unchanged tracer in plasma ( $f_p$ ), i.e. the fraction of tracer not bound to plasma proteins. Often  $f_p$  is difficult to measure and thus not explicitly used. Hereby,  $f_p = 1$  is assumed, and the unchanged tracer concentration is regarded as the input function.

After the different tracer fractions in blood have been separated, it is necessary to calculate their activity concentrations. This is done by measuring them in a radioactivity counter and dividing the activity by the fractional volumes.

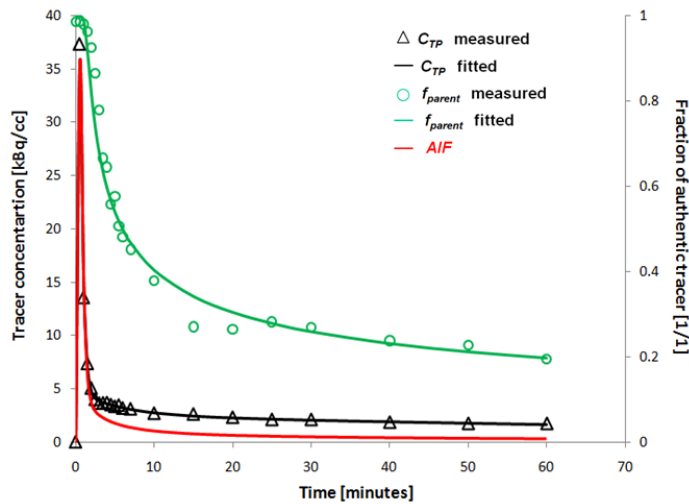
### Fitting of Blood Curves by Models

Due to the experimental procedures involved the blood activity measurements are typically noisy and contribute a considerable amount of uncertainty to the analysis. Therefore it is often reasonable to fit analytical functions to the blood-related measurements, which is also applied for the interpolation between the samples, as well as for the extrapolation at late times when blood samples might be lacking.

The example below illustrates the blood model approach.

- ▶ The black triangles represent the activity of tracer (free, bound to proteins, metabolites) in the arterial plasma during the study. A tri-exponential function which was fitted to these measurements and shown as the black line.
- ▶ The green circle represents the measurements of the fraction "Unchanged tracer in plasma" to "Total tracer in plasma". A model was fitted to these fraction values and shown as the green line.
- ▶ The arterial input function (AIF) is obtained by the multiplication of the two model functions and represented by the red curve, implicitly assuming  $f_p = 1$ .

Note the sharp concentration changes at the early times after injection. To capture these changes adequately, rapid blood sampling is required during the first minutes after tracer injection.



## Calculation of the Model Curve

The signal of a dynamic PET measurement represents the averaged activity in the image pixels at acquisition times  $t$  starting at tracer injection. It is described by the **operational equation**

$$C_{Model}(t) = (1 - vB) C_{Tissue}(t) + vB C_{Blood}(t) \cong C_{PET}(t)$$

This equation means that the activity concentration  $C_{PET}$  measured by PET in a certain tissue volume is composed of two contributions:

- ▶ tracer which has been extracted into tissue, where it has an instantaneous concentration of  $C_{Tissue}(t)$ .
- ▶ tracer which is circulating within the blood, with a concentration  $C_{Blood}(t)$ .

Hereby it is assumed, that the fractional volume  $vB$  is composed of small capillaries (2%-5%) filled with whole blood, and that the fraction  $(1-vB)$  represents tissue. The blood activity  $C_{Blood}$  must be measured during the acquisition. In tissue, the tracer may be present in different spaces or forms (eg. *free*, *specifically bound*, *non-specifically bound*), which are

described by the compartments of the model. All compartments contribute to the tissue signal, so that it is modeled by the expression

$$C_{Tissue}(t_k) = \sum_i \frac{1}{(t_k^{end} - t_k^{begin})} \int_{t_k^{begin}}^{t_k^{end}} C_i(t) dt$$

whereby

- ▶  $C_{Tissue}(t_k)$  represents the average concentration of tracer in tissue during the acquisition  $k$  which starts at  $t_k^{begin}$  and lasts until  $t_k^{end}$
- ▶  $C_i(t)$  represents tracer concentration in compartment  $i$  at time  $t$ . These expected concentrations are calculated from the differential equations using the current model parameters and the plasma input curve(s).

The operational equations used for other than compartment models are specified in the PKIN Model Reference *section* (on page 107).

## Fitting and Residual Weighting

### Least Squares Optimization

The fitting methods available optimize the agreement between the measurements and the model curve. Effectively, they minimize the difference between them, whereby the difference is described by the Chi Squares criterion (cost function) below:

$$\chi^2 = \sum_i w_i [C_{PET}(t_i) - C_{Model}(t_i)]^2$$

This expression implies that the squared residuals (measured value minus estimated model value) are multiplied by weights. To satisfy the requirements of *least squares* fitting, the weights  $w_i$  should be related to the variance  $\sigma_i^2$  of the measurements by

$$w_i = \frac{1}{\sigma_i^2}$$

In this case, and provided that the distributions of the measurement error are normal, the estimate obtained is the *maximum likelihood* estimate. Under the same premise it is also possible to obtain standard errors of the model parameters as the square root of the diagonal elements in the covariance matrix.

### Weighting of PET Data

The variance of reconstructed PET data is dependent on many factors, including the duration of the acquisition, the time since the scan start which needs to be compensated by a decay correction, scatter and random correction, the sampling volume, the reconstruction method, etc. Therefore, PET variance models used in weighting of the residuals during fitting are always approximations.

In his presentation *Parameter Estimation: Least squares and why it gives you fits* (Handouts of Pharmacokinetic Course 2009) Richard E. Carson gives the following hints on using weights:



- ▶▶ Uniform weights (ordinary least squares fitting): Even if the data does not have uniform variance the estimates should be unbiased, but the parameter standard errors will be higher than they could be.
- ▶▶ Wrong weights: The greater the error in the weights, the larger the loss of precision in the parameter estimates (more important for  $^{11}\text{C}$  than  $^{18}\text{F}$ ).

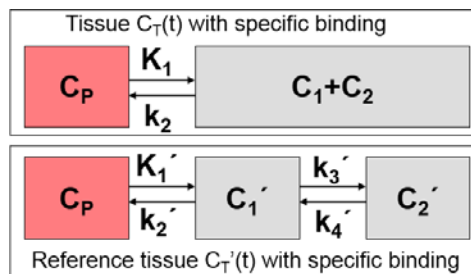
## Reference Tissue Models

The measurement and analysis of the blood samples for kinetic modeling is an invasive and demanding procedure. Therefore, methods have been developed to obviate the need for invasive blood sampling. The solutions found replace the arterial input curve by an indirect input curve, namely the time activity curve of some reference tissue. Therefore they are called *reference methods*. Reference methods are not able to provide a full kinetic analysis. However, assuming certain relations between the kinetics of the tissue of interest and the reference tissue, they can provide valuable measures of interest.

Most of these reference methods are dedicated to reversibly binding neuroreceptor tracers. A reference tissue must be found which is devoid of receptors, and then it is assumed that the distribution volume of the non-displaceable compartment (free tracer in tissue and non-specific binding) is the same among the tissues. Under these assumptions a measure of the receptor concentration called *binding potential* can be calculated from the two time-activity curves.

### Model Structure

The compartment models are usually based upon the following configuration:



In the model equations  $C'(t)$  represents the TAC from the reference region ( $k_3=0$  in the 2-tissue compartment model), and  $C(t)$  the TAC from a receptor-rich region ( $k_3>0$ ).

However, the various reference methods differ in their mathematical approaches, and they show substantial differences with regard to noise sensitivity and processing speed. They are described in the *PKIN Model Reference* (on page 142) section.

## Determination of $k_2'$ for Models Assuming a Pre-determined $k_2'$ Value

Some of the reference methods require an a priori average value of  $k_2'$ , while other methods such as the MRTM or the SRTM reference methods estimate  $k_2'$  together with the other parameters.

### Averaging $k_2'$ from Separate Regional Fits

In response to a request from a PMOD user Dr. Ichise recommended the following approach for calculating  $k_2'$ :

- 1)  $k_2'$  is the tissue clearance rate from the reference region, e.g., cerebellum. This reference tissue is always a region (ROI), not a voxel for both ROI based PKIN or PXM0D

parametric imaging. If you define a reference region, there should be only one correct  $k_2'$  value for that particular subject (scan).

Logan in her original formulation of her reference tissue model suggested to determine  $k_2'$  by using arterial data for a group of subjects and use this mean  $k_2'$ .

However, we showed that this  $k_2'$  can be estimated for each individual without arterial data using MRTM or SRTM (three parameter estimation, one of the three parameters is  $k_2'$ ).

I prefer to use  $k_2'$  estimated this way for each subject for the subsequent MRTM2 or SRTM2.

This would be more accurate than the mean  $k_2'$  estimated for a group of subjects as above.

Now the accuracy of  $k_2'$  estimation depends on the following (see [52]): A) noise in the PET data, B) the magnitude of  $k_2'$  and C) the ratio of  $k_2'/k_2$  or  $k_2/k_2'$  determined by MRTM or SRTM.

The bias and variability of  $k_2'$  estimation by MRTM is less as  $k_2'$  is larger and  $k_2'/k_2$  or  $k_2/k_2'$  ratio is further away from unity. This ratio for fallypride using cerebellum and striatum should be greater than 3, I think. In that case,  $k_2'$  estimation from cerebellum and striatum (use ROIs) should be minimum (see figs in the paper).

Even using the ROIs,  $k_2'$  estimation is affected by noise and hence it is good to run MRTM a few times choosing ROIs with high  $BP_{ND}$  areas (say right striatum and left striatum)  $k_2'/k_2$  ratio is further away from 1) and average the  $k_2'$  values. This averaging is within the subject and totally different from population average.

Please use the  $k_2'$  determined as above for estimation of  $BP_{ND}$  for the cortical regions. The  $k_2'$  estimated with cerebellum and cortical regions is not accurate because  $k_2'/k_2$  is closer to unity.

One advantage of MRTM over SRTM: To use SRTM, both cerebellum and target must be 1 tissue (1T) kinetics (use of the SRTM for 2T will bias  $BP_{ND}$ ). However, MRTM is good for tracers with 2T kinetics such as Fallypride. The only thing here is that you have to give a  $t^*$  value.

### Estimating $k_2'$ using a Fit with Regional Coupling

Alternatively, the a-priori knowledge that  $k_2'$  should always have the same value can be exploited using *coupled fitting* (on page 72) in PKIN with SRTM2 as follows:

- 1) Select the SRTM2 model for all regions and fit it with  $k_2'$  enabled.

Parameter	Current value	Unit	% SE
<input checked="" type="checkbox"/> R1	0.989974	1/1	2.82
<input checked="" type="checkbox"/> $k_2'$	0.066829	1/min	6.96
<input checked="" type="checkbox"/> $k_{2a}$	0.02712	1/min	5.47

This will result in regionally different  $k_2'$  values which should be quite similar across the high-binding regions.

Configure a coupled fit with  $k_2'$  as the common parameter and only include the relevant regions for coupling.

Coupled parameter	Current value	Unit
<input type="checkbox"/> R1	0.989974	1/1
<input checked="" type="checkbox"/> $k_2'$	0.066829	1/min
<input type="checkbox"/> $k_{2a}$	0.02712	1/min
sampl. interval	5.0	sec
Coupled region		
<input type="checkbox"/> cerebellum		
<input type="checkbox"/> temporal cortex		
<input checked="" type="checkbox"/> caudate		
<input checked="" type="checkbox"/> putamen		
<input checked="" type="checkbox"/> thalamus		
<input type="checkbox"/> visual cortex		
<input type="checkbox"/> parietal cortex		
<input type="checkbox"/> frontal cortex		

Start the coupled fit which returns a common  $k_2'$ . This value can be used for pixel-wise fitting or the regional fitting of all TACs

For the regional fitting fix  $k_2'$  in one of the coupled regions

Parameter	Current value	Unit	% SE
<input checked="" type="checkbox"/> R1	1.06399	1/1	1.66
<input checked="" type="checkbox"/> $k_2'$	0.053937	1/min	3.95
<input checked="" type="checkbox"/> $k_{2a}$	0.022882	1/min	3.69

propagate the model to all regions, and then fit all regional TACs.

The same  $k_2'$  value can also be used for pixel-wise fitting with the PXMOD tool.

## Spectral Analysis (SA)

The closed-form solutions of compartment models involve the convolution of the input function with decaying exponentials. Based on this observation, a generalized technique called spectral analysis (SA) was introduced by Cunningham and Jones [66]. The operational equation of SA is given by

$$C_{PET}(t) = \sum_{i=1}^N a_i C_p(t) \otimes e^{-\beta_i t} = \sum_{i=1}^N a_i B_i(t)$$

that is, tissue uptake is modeled as a sum of  $N$  possible tissue responses. Due to the constraint of first order tracer kinetics, the coefficients  $a_i$  and the decay constants  $\beta_i$  must be non-negative. In practice, a discrete set of the decay constants  $\beta_i$  is selected which covers the physiologically reasonable range, typically logarithmically spaced in the range  $[10^{-5}, 1] \text{sec}^{-1}$ . The corresponding tissue responses

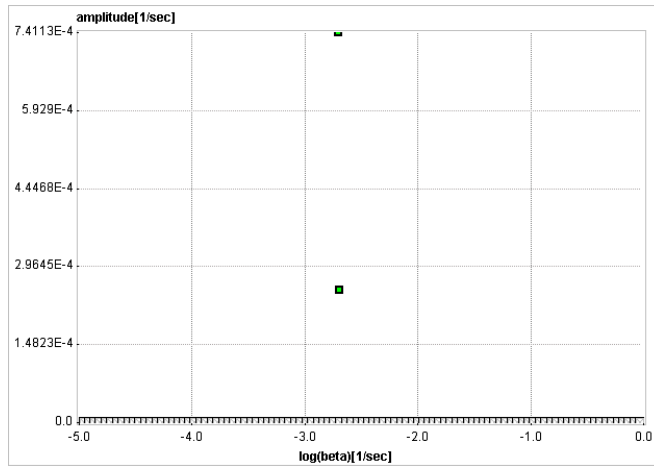
$$B_i(t) = C_p(t) \otimes e^{-\beta_i t}$$

are the basis functions of spectral analysis.

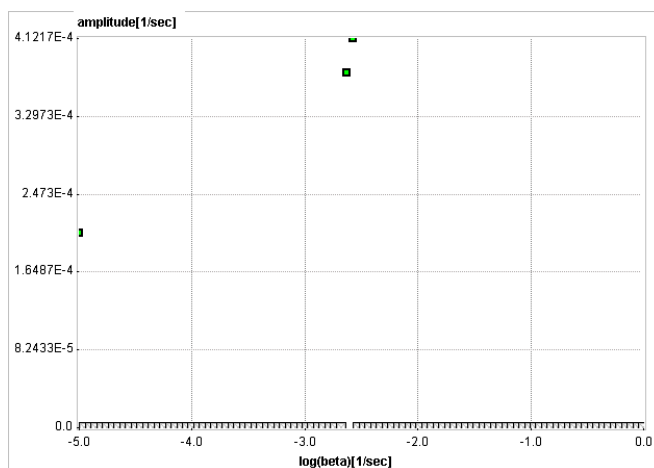
When fitting the operational equation above to a tissue TAC, the only unknowns are the coefficients  $a_i$ , because only a pre-defined set of discrete  $\beta_i$  values is considered. Therefore, the problem is that of a non-negative linear least squares estimation (NNLS) with the constraint of non-negative coefficients. There is a well-known NNLS algorithm available [67], which allows readily calculating the optimal set of  $a_i$  coefficients. They allow calculating the model function and visualizing it together with the tissue TAC in the same way as for the compartment models.

An advantage of SA is the fact that no particular compartment structure is imposed. Rather, its result can be used to estimate how many kinetic tissue compartments can be resolved by PET. To this end, the results are plotted as a spectrum with the selected decay constants  $\beta_i$  along the x-axis (as the "frequencies") and the estimated coefficients  $a_i$  along the y-axis (as the "amplitudes"). Because of the large range,  $\log(\beta_i)$  is used in spectrum plotting rather than  $\beta_i$ . The number of peaks in this spectrum corresponds to the number of distinct compartments. A peak appearing to the far left (low frequency, slow component) indicates irreversible trapping. A peak to the far right (high frequency, fast component) corresponds to kinetics indistinguishable from the input curve, thus to vascular contributions. Intermediate peaks represent compartments which exchange reversibly with plasma or with other tissue compartments [68].

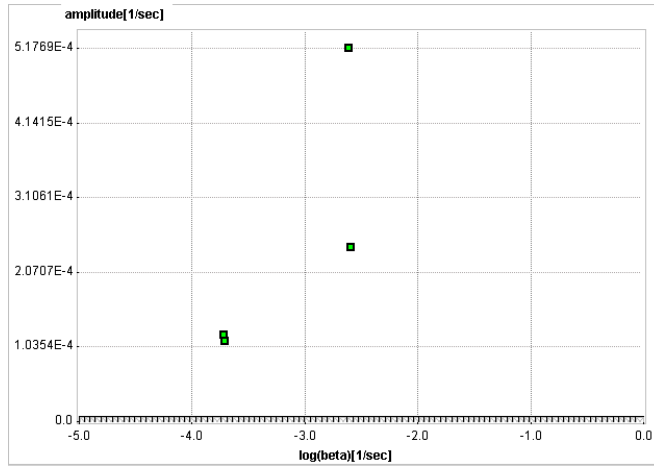
The examples below used synthetic data generated using compartment models without noise and blood contributions. In the first example the simulated tissue TAC of a 1-tissue compartment model was fitted by SA with 500 basis functions. It is evident that, because of the discrete nature of the  $\beta_i$  basis, the compartment was split into two neighboring frequencies. The adjacent amplitudes are usually summed to provide the combined peak height for a compartment.



In the next example an irreversibly trapping compartment was added ( $k_3 > 0$ ,  $k_4 = 0$ ) to the same first compartment. The corresponding peak is clearly seen to the left of the spectrum.



When changing kinetics to become reversible ( $k_4 > 0$ ), the second compartment appears as a second peak in the inner of the spectrum.



In general it is not possible to calculate the compartment rate constants from the spectral analysis outcome. However, an estimate of  $K_1$  can be obtained as the sum of the peak amplitudes

$$K_1 = \sum_{i=1}^N a_i$$

and an estimate of the distribution volume  $V_T$  as

$$V_T = \sum_{i=1}^N \frac{a_i}{\beta_i}$$

An additional information which can be calculated is the impulse response function

$$IRF(t) = \sum_{i=1}^N a_i e^{-\beta_i t}$$

The impulse response function completely describes the system, and the expected tissue TAC for any given input function and can simply be calculated by convolution with the IRF:

$$C_{PET}(t) = C_P(t) \otimes IRF(t)$$

In the practical application of spectral analysis potential problems were found: Caution should be applied when interpreting the number of peaks [66], and the error properties of the estimates are difficult to assess [69]. Several authors have proposed variants to overcome these problems [70,71], but in current practice spectral analysis is not yet frequently applied.

## Chapter 1

# PKIN Configuration

There are two panels where the functionality and layout of PKIN can be configured. The available models are managed on the general configuration panel, the layout options with the local configuration button of the PKIN tool. Additionally, the list of shown models can be fine-tuned using the *filtering* (on page 62) functionality.

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## Global Configuration with Config Button

Use the



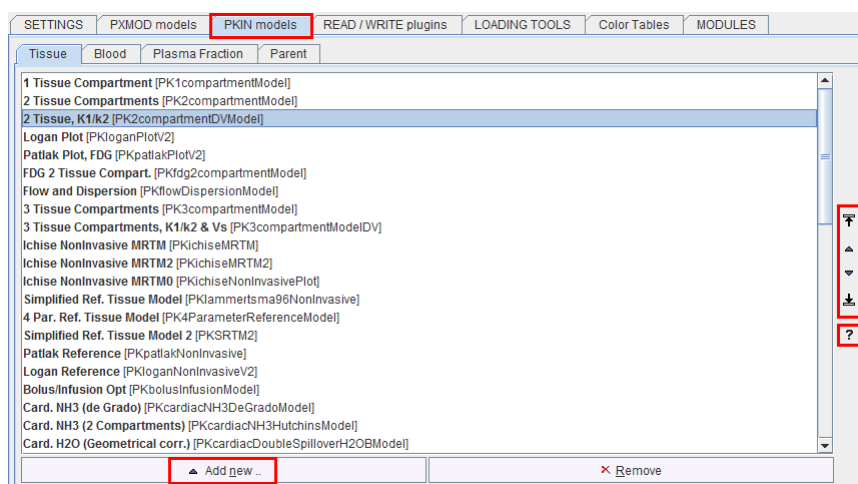
on the PMOD ToolBox to show the general configuration dialog.

Many PMOD functionalities are programmed as plug-ins. Most of these plug-ins are initially installed, but they can be removed or rearranged for each user account with the different tabs. The procedure is always the same and is explained with the plug-ins (the models) of the kinetic modeling tool PKIN.

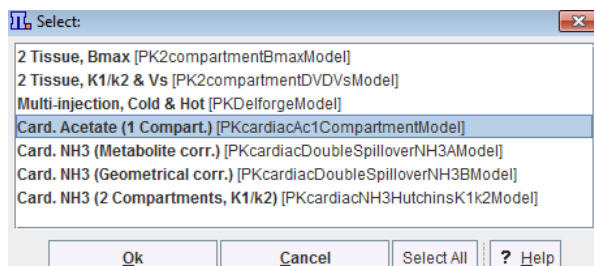
### PKIN Models

When the **PKIN models** tab is selected, the list of the currently installed model plug-ins is shown. The entries show the name in the model selection of the PKIN tool as bold and in parentheses the file names of the plug-in.

To modify the order how the models appear in the PKIN application please select an entry and move it up/down using the arrows to the right. The ? button shows a quick model explanation.



To remove a model from the list just select its list entry and activate **Remove**. To add a model back to the list use the **Add new** button. The appearing dialog window allows selecting one or more plug-in(s) and adding them by the **Ok** button.



## Contents of the different Plug-In Tabs

<b>PXMOD models</b>	Configuration of the models which appear in the pixel-wise modeling (PXMOD) tool.
<b>PKIN models</b>	Configuration of the models which appear in the modeling tool for time-activity curves (PKIN). <b>Tissue</b> contains the actual kinetic models for the tissue TACs, <b>Blood</b> the models for interpolation of the blood activity, <b>Plasma fraction</b> the models for plasma fraction activity and <b>Parent</b> the models for metabolite correction.
<b>READ/WRITE plug-ins</b>	Configuration of the image data file formats. Note that only a subset of the formats can be written.

```

PMdynamicFormatAnalyze
PMdynamicFormatA/W/Analyze
PMdynamicFormatDBase
PMdynamicFormatDicom
PMdynamicFormatDicomCStore
PMdynamicFormatEcat
PMdynamicFormatGEAdvanced
PMdynamicFormatGraphic
PMdynamicFormatHidacPet
PMdynamicFormatInterfile
PMdynamicFormatMatlab
PMdynamicFormatMicroPET
PMdynamicFormatRaw
PMdynamicFormatTiff

```

<b>LOADING TOOLS</b>	Configuration of image processing filters usable during loading.
----------------------	------------------------------------------------------------------

<b>Color Tables</b>	Configuration of user-defined color tables. These files must be
---------------------	-----------------------------------------------------------------

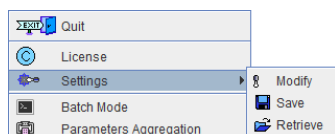
- » located in the resources/colortables sub-directory (see the examples there),
- » text files ending in .cltb, and
- » contain 3 columns with the RGB values such as
 


```
# R G B
0 0 0
0 2 2
0 4 4
etc.
```

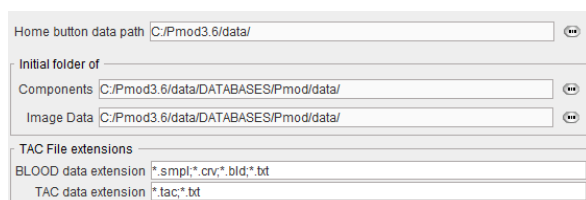
System color tables cannot be removed or arranged in a different order.

## Local PKIN Configuration

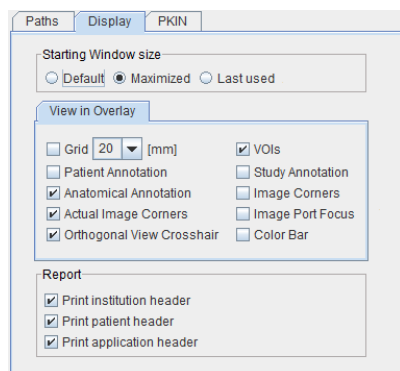
The configuration of PKIN can be adjusted by **Kinetic/Settings/Modify**



or using the configuration button  from the status line. It opens the following dialog with the current definitions, which can be modified. The **Paths** tab allows setting the initial data paths of file-based loading as well as the file extension filters.



The **Display** pane gives access to the settings of the initial PKIN appearance and the appearance of the report page.



The **PKIN** pane shows the current models of tissue, whole blood, plasma and the parent fraction with their respective default parameters. Changing the parameters here and saving will establish new default parameters.

The next section allows specifying the time and value units of data loaded from text files. If the header lines in the data file do not contain valid units, these units are applied to the imported data.

Parameter	Value	Restrict	Lower	Upper	Search	Value	Units
vB	0.05	<input checked="" type="checkbox"/>	0.0	1.0		0.01	1/1
K1	0.1	<input checked="" type="checkbox"/>	0.0	Upper 8.0	) Search	0.1	ml/ccm/min
k2	0.1	<input checked="" type="checkbox"/>	0.0	Upper 8.0	) Search	0.1	1/min
k3	0.01	<input checked="" type="checkbox"/>	0.0	Upper 8.0	) Search	0.1	1/min
k4	0.01	<input checked="" type="checkbox"/>	0.0	Upper 8.0	) Search	0.1	1/min

Blood TABULATED Units (value, time) kBq/cc seconds

Tac TABULATED Units (value, time) kBq/cc seconds

Curve controls position Right

Center model parameters display ☐

HD curve model display ☒

The lower section serves for modifying the layout:

- » **Curve controls position** allows displaying the curve controls to the **Right**, **Left** or at the **Bottom**. Their purpose is to optimize the layout depending on the display aspect ratio. **Left** and **Right** are recommended for wide screen systems, otherwise **Bottom**.
- » **Center model parameters display**: If this box is enabled the parameters area is centered vertically within the available space.
- » **HD curve model display**: If this box is checked the model curves are calculated for display purposes at many intermediate points, not just at the measurement times. The result is a smooth appearance of the model curves. However, when exporting the model curves, these interpolated points may be disturbing. Note that for the fitting only the original acquisition times are used.

## Chapter 2

# PKIN Data Processing

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## General Assumptions in PKIN

The following is generally assumed:

- ▶▶ All loaded data has been **decay corrected** to the same time point. This means that the PET scanner and the blood sampling times must have been synchronized.
- ▶▶ The acquisition time intervals may have gaps, but must not overlap.
- ▶▶ The **input curve**  $C_p(t)$  represents the authentic tracer in arterial plasma which is available for exchange with tissue.
- ▶▶ The **whole blood** curve  $C_{\text{Blood}}(t)$  represents the activity concentration of the tracer and all its metabolites in whole blood.
- ▶▶ In compartment models the whole-blood signal contribution can optionally be considered in the operational equation by
 
$$C_{\text{Model}} = v_B * C_{\text{Blood}} + (1 - v_B) * C_{\text{Tissue}}$$
 where  $v_B$  represents the fractional volume of the blood space in the VOI,  $C_{\text{Blood}}(t)$  the whole blood concentration, and  $C_{\text{Tissue}}(t)$  the summed concentrations in all tissue compartments.
- ▶▶ The results of non-cardiac flow models are returned per  $\text{cm}^3$  tissue. To convert to flow per g tissue the values must be divided by the tissue density.

Exceptions to these rules are specified in the description of the individual models.

## Processing Overview

Data processing of studies with PET or SPECT tracers typically consists of the following parts:

<b>1. Data Loading</b>	<ul style="list-style-type: none"> <li>» Create a new data set.</li> <li>» Load the whole blood curve (total activity concentration of tracer in the blood samples).</li> <li>» Load the plasma <i>activity</i> curve. Depending on the preparation of the blood data the plasma activity curve can represent the unchanged tracer (parent) in plasma, or total tracer in the plasma of the blood samples including the metabolites. In the latter case of total tracer the metabolite correction has to be performed in PKIN.</li> <li>» An alternative to loading the plasma activity curve is loading the plasma <i>fraction</i> curve. In this case the total plasma activity is calculated by multiplying whole blood activity with the plasma fraction.</li> <li>» Load the parent fraction curve (fraction of unchanged tracer in plasma). This is only necessary if the loaded plasma curve represents total tracer activity in plasma including metabolites.</li> <li>» Load the average time-activity curves of one or multiple tissue regions.</li> </ul>
<b>2. Input Curve Configuration</b>	<ul style="list-style-type: none"> <li>» Define interpolation functions (called models) for the different types of blood data and fit them to the data. The default is linear interpolation between measurements, and a parent fraction identical to 1.</li> <li>» The input curve is obtained as the multiplication of the plasma model with the parent fraction model.</li> </ul>
<b>3. Kinetic Model Configuration</b>	<ul style="list-style-type: none"> <li>» Select a region with a "typical" time-activity curve (TAC) from the <b>Region</b> list.</li> <li>» Select a simple model from the <b>Model</b> list.</li> <li>» Enable the parameters to be fitted by checking their boxes; unchecked parameters remain at the initial values entered.</li> <li>» Select a weighting scheme of the residuals; variable or constant (default) weighting is available.</li> </ul>
<b>4. Kinetic Model Fitting</b>	<ul style="list-style-type: none"> <li>» Fit the model parameters by activating the <b>Fit current region</b> button.</li> <li>» Consult the residuals to check whether the model is adequate; there should ideally be no bias in the residuals, just random noise.</li> <li>» If the model fit is fine, configure <b>Copy the model to all regions to Model &amp; Par</b> and activate the button to establish the same initial model configuration for all TACs. Then activate <b>Fit all regions</b> to adjust the parameters to the regional measurements.</li> <li>» Check the fit result for all the TACs.</li> <li>» If the model is not yet fine, test more complex models.</li> </ul>
<b>5. Kinetic</b>	<ul style="list-style-type: none"> <li>» Switch between compartment models of different complexity and</li> </ul>

<b>Model Comparison</b>	<p>fit. The parameters are either maintained for each model type, or converted, according to the <b>Model conversion</b> setting in the <b>Extras</b> panel.</p> <ul style="list-style-type: none"> <li>» The results of the different fits are stored in the <b>Model History</b> which can be used for easy switching and for the results comparison per region.</li> <li>» Check the residuals for judging model adequacy.</li> <li>» Check the different criteria on the <b>Details</b> tab (Schwartz Criterion <b>SC</b>, Akaike Information Criterion <b>AIC</b>, Model Selection Criterion <b>MSC</b>) to decide whether a more complex model is supported by the data.</li> <li>» Check for parameter identifiability. As an indicator of the parameter identifiability standard errors (%SE) are returned from the fit. They should remain limited for all relevant parameters. Additionally, Monte Carlo simulations can be performed to obtain distribution statistics of the parameter estimates.</li> <li>» If justified by physiology, try to improve the stability of parameter estimation by enforcing common parameters among regions in a coupled fitting procedure.</li> <li>» Compare the outcome of compartment models with that of other models, such as reference models or graphical plots.</li> </ul>
<b>6. Saving</b>	<ul style="list-style-type: none"> <li>» Save all model information together with the data in a composite text <b>.km</b> file which allows restoring a session including the fitting history.</li> <li>» Save a summary of all regional parameters in an EXCEL-ready text file <b>.kinPar</b>.</li> </ul>
<b>7. Batch Processing</b>	<p>Lengthy calculations such as coupled fits with many regional TACs or Monte Carlo simulations can be run in a batch job. The results are saved both as <b>.km</b> files and in an EXCEL-ready text file.</p>



# Starting the Kinetic Modeling Tool

The kinetic modeling tool is started with the **Kinetic** button from the PMOD Toolbox

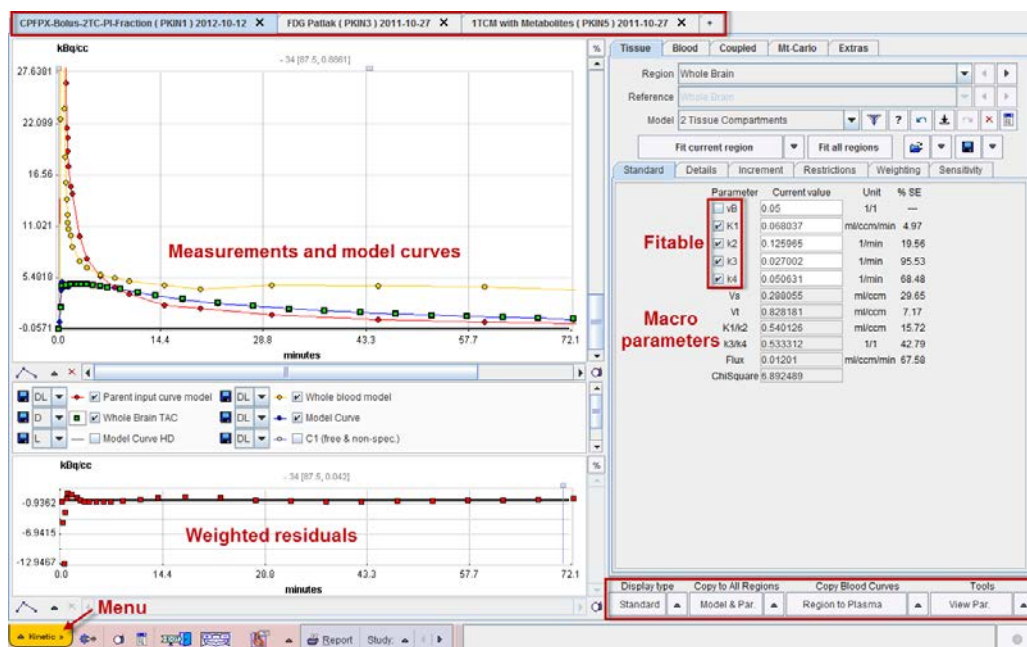


or by directly dragging kinetic modeling (.km) files onto the above button.

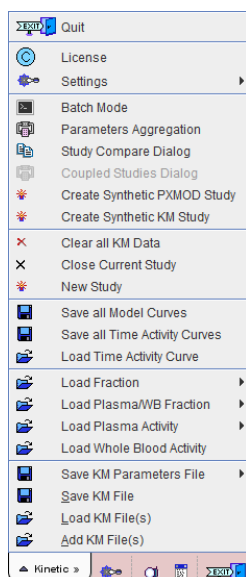
The user interface is organized as illustrated below. The left part of the display visualizes the data, the model and the fit. The green squares represent the tissue measurement values (tissue time-activity curve, TAC), the red circles the input curve, the yellow circles the blood spillover curve (whole blood time-activity curve), and the blue line the calculated model curve with the current model configuration shown to the right. All curves can be shown or hidden using the check boxes in the control area.

The lower curve display shows the weighted residuals, i.e. the difference between the measurement and the model curve multiplied by a weighting factor. Unweighted residuals can also be shown.

The right part gives access to the different models and operations which are described in the following sections.



The **Kinetic** menu is located in the lower left corner and is used for data operations such as loading, saving or closing.



Loaded data sets can be added into new tabs, so multiple data sets can be available simultaneously and selected for processing using the named tabs.

## Data Loading/Saving

There are different model configurations which may require both blood and tissue data, or only tissue data.

### Tissue Activity Data

Tissue TAC data can most easily be brought into PKIN by outlining VOIs in the PVIEW tool and sending average time-activity values in the different VOIs directly to PKIN. This approach has the advantage that the standard deviations within the VOIs, demographic and activity-related information are also transferred. As an alternative, the TAC information can be prepared in a text file and loaded.


### Blood Data

The blood data is typically the result of taking wet blood samples, plasma separation, metabolite analysis and activity counting in a separate device. In these cases the measurement results must be arranged properly in a text file and loaded. If the separation of tracer metabolites in blood is not necessary or can be achieved by multiplication with a correction function (parent fraction), and blood activity can be derived directly from the images (eg. in cardiac studies), it is also possible to use the VOI approach.

### Composite .km File

Loading and processing of a study results in a workspace consisting of measured data and various model configurations. Such a whole workspace can be saved in a comprehensive text file with suffix **.km**. When loading a .km file the state of the previous processing including the regional fitting history is fully restored with the exception of the display settings.

### Data Organization

Several data sets can be processed in parallel. They are shown on separate pages in the PKIN tool, with selection through the upper tabs. The **New study** entry in the **Kinetic** can be used for creating a new empty data set, to which the blood and tissue measurements can be loaded. The same result can be obtained by activating the  tab. The **Load** entries *overwrite* the information in the currently selected data set. With **Add KM file** a complete study can be loaded into a newly created tab without overwriting existing data.

## File-based Data Import

There are slightly different formats for the time-activity data of blood and tissue. With blood data it is assumed that instantaneous sampling was performed at well-defined time points. With tissue data it is assumed that the measurements represent the average tracer concentration during a certain observation period, the PET frame duration defined by start-time and end-time.

### Data Units

Typically the data will arise in the following units:

- ▶▶ [seconds], [minutes] or [hours] for the time specification, and
- ▶▶ [kBq/cc], [uCi/cc] or [1/1] for the value specification.

The unit string in square brackets should immediately follow the column header. If units are not defined in the column headers or not recognized, it is assumed that the units specified in the PKIN *configuration* (on page 26) apply.

---

**Note:** During loading, the input unit specification will be used to **convert the activity values into the internal representation of kBq/cc**. All loaded data will be displayed with these values after conversion.

---

## Import of Blood Data

PET tracers are usually applied by intra-venous injection and then brought to the tissue of interest by circulation. Often, a part of the tracer may be bound to red blood cells. Another part may be processed in organs and end up as labeled metabolites circulating in blood plasma. The remaining category of tracer in plasma is the unchanged (authentic) tracer (parent) which is available for exchange with tissue and represents the *input curve* relevant for modeling.

In PKIN, the following four types of blood data are supported to model the contributions of the different forms of tracer in blood to the expected PET signal:

- 1) The concentration of all forms of tracer in **Whole Blood** samples: As the average signal from a VOI always contains a fraction of signal from blood vessels and capillaries, the concentration in whole blood is needed for modeling the blood content in the expected PET signal.

The concentration of all forms of tracer in plasma (**Plasma Activity**) : Although the unchanged tracer in plasma is required as the input curve of the models, it is common practice to measure the tracer activity of the whole plasma sample.

The fraction of unchanged ligand in plasma (**Parent Fraction**): As a result of the metabolite analysis of plasma samples, the relative concentrations of the unchanged tracer and the metabolites are known. This information is represented by the ratio of unchanged tracer to total tracer. This fraction is expected to start with a value of 1 at the time of injection (all tracer is unchanged), and gradually decreases as the metabolites build up. Since plasma analysis is often experimentally complex and therefore error-prone, it is advisable to fit a smooth curve to the measured parent fraction.

The **Plasma Fraction**: An alternative to using the measured plasma activity is the use of the ratio of plasma activity to whole blood activity, called the "Plasma Fraction". If the plasma fraction is known it can be multiplied with the whole blood activity to obtain the plasma activity. For tracers without metabolites this obviates the need for the actual blood analysis.

## Whole Blood and Plasma Activity

The whole blood and plasma activity concentrations must be prepared in text files, and can be loaded using **Load Whole Blood Activity** and **Load Plasma Activity** from the **Kinetic** menu. Such text files with the blood data can be prepared for example in MS Excel and then saved as tab-delimited or csv separated text files. There are two variants of the format:

- 1) Separate files for whole blood and plasma activity. In this case separate files are prepared with a header line, the sample time in the first column and the sample value in the second column as illustrated below:

sample-time[seconds]	plasma[kBq/cc]
0	0
6	0.01
12	0.02999997
18	37.94988615
24	118.4692892
30	138.4187542
36	96.68874303
42	68.22877186
48	55.90871407
54	38.498845

sample-time[seconds]	whole-blood[kBq/cc]
0	0
6	0
12	0.02
18	22.64
24	71.19
30	81.84
36	52.8
42	41.62
48	33.72
54	23.76

A composite file containing both whole blood and plasma activity. Note that in this case the keywords **sample-time**, **plasma** and **whole-blood** are required to define the meaning of the columns.

sample-time[seconds]	plasma[kBq/cc]	whole-blood[kBq/cc]
0	0	0
6	0.01	0
12	0.02999997	0.02
18	37.94988615	22.64
24	118.4692892	71.19
30	138.4187542	81.84
36	96.68874303	52.8
42	68.22877186	41.62
48	55.90871407	33.72
54	38.498845	23.76

## Plasma and Parent Fractions

The plasma and parent fractions need to be prepared in a similar file

time[minutes]	parent-fraction[1/1]
0	0.27184857
0.5	0.986298884
1	0.980287666
1.5	0.96249071
2	0.92520486
2.5	0.865100226
3	0.779710538
3.5	0.666156902
4	0.645169911
4.5	0.558181271
5	0.577178078
5.5	0.50647541
6	0.480742985

and loaded using **Load Plasma/WB Fraction** or **Load Fraction/Parent** from the **Kinetic** menu.

### Notes:

1. If no whole blood data is loaded into PKIN, the plasma concentration is used for blood spillover correction.
2. If the activity of unchanged tracer in plasma is loaded instead of the total plasma activity, no correction with the parent fraction is required. No further action is required in this case, because per default it is assumed that the parent fraction equals the constant of 1.
3. There are models which require two input curves, and in principle models with up to 10 input curves can be handled in PKIN. For these models the loading sub-menus contains appropriately labeled entries.

## Import of Tissue Time-Activity Data

Tissue time-activity curves are similarly organized in tab-delimited text files. The frame start and end times are arranged in the first two columns. Next come columns with the measurements from different tissues. Note that the TAC value units are specified after the heading of the *second* column.

start[seconds]	end[kBq/cc]	amy re	amy li	mtl re
0	60	2.29234465	1.98046206	2.74814255
60	120	5.01681393	6.57439037	6.81311774
120	180	6.18648905	5.47218649	6.87476477
180	240	6.57449309	6.58147725	7.33382369
240	300	5.95458466	6.59546425	7.65397945
300	360	7.06336248	7.3919664	7.21261065
360	420	7.07207865	8.066492	6.83445489
420	480	6.44191169	7.89309654	7.34035093
480	540	5.73063921	5.38436545	7.231256
540	600	6.35527845	7.03980869	7.31066749
600	900	6.04079722	6.51268169	6.96765792

Tissue time-activity curves are loaded using **Load Time Activity Curve** from the **Kinetic** menu.

## Composite Import Format for all Data

The composite format allows including all types of tissue and blood data into a single tab-delimited text file. The advantage is that all data can be prepared and loaded at once, and that such files can directly be used for batch processing.

### File Start

The file starts with the two lines

```
# APPLICATION
KINETIC_MODELING_DATA
```

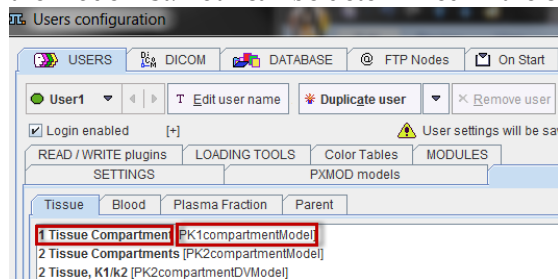
It is followed by an optional model specification and a sequence of data sections.

### Model Specification

A tissue model can be specified as follows.

# MODELS			
// ID	Model	Name	
KINETIC	PKepideptideModel	3_Comp._Metabolites	

The **Model** entry corresponds to the class name, and the **Name** entry to the string shown in the model list. Both can be determined in the configuration utility as illustrated below.



For reference models the reference region can be specified in by an additional line with the **REFERENCE\_REGION** tag.

# MODELS			
// ID	Model	Name	
KINETIC	PKichiseMRTM	Ichise Reference MRTM	
REFERENCE_REGION		1 KH re	

Note that the region number (starting from 0) is relevant, not the region name.

## Data Sections

Each data part to be included is specified in a section starting with a line #DATA followed by a header line of the actual data, for instance in the form of

# DATA <b>Section start</b>	
// sample-time[minutes]	whole-blood[kBq/cc]
0.75	1.67E-05
1.5	4.818734894
2.25	18.47443512
3.02	21.04629028
3.75	27.0131493
4.52	12.99452924
5.25	8.306559731
6.1	5.448897549
7.98	3.980082036
10.07	3.663062688

for blood data and

# DATA			
// start[seconds]	end[kBq/cc]	tac.Cerebellum	tac.Putamen
0	30	0.411745285	0.321251619
30	60	7.758802694	10.87512199
60	90	18.56423739	21.30137194
90	120	27.93297721	39.70225133
120	150	37.67981055	56.65117912

for tissue data. Note that the columns are separated by tabs.

The header line contains the column headers with the units of the data column in brackets. The following column headers are keywords and determine the use of the data values. The supported keywords are:

<b>sample-time</b>	Timing of blood samples.
<b>whole-blood</b>	Activity of whole blood samples, to be used for spillover correction. May also be used for calculating the plasma activity if a plasma fraction is loaded.
<b>plasma.1</b> <b>plasma.2</b>	Plasma activity of the first input curve, the second, etc. Plasma activity may represent the activity of parent, if the metabolite correction has been done outside of PKIN, otherwise the activity of parent and metabolites. In the latter case a parent fraction has to be loaded for the metabolite correction within PKIN.
<b>plasma-fraction.1</b> <b>plasma-fraction.2</b>	Plasma fraction of the first input curve, the second, etc. These fractions will be multiplied with <b>whole-blood</b> for the calculation of <b>plasma.1</b> , <b>plasma.2</b> etc. Supported units: [1/1], [%].
<b>parent-fraction.1</b> <b>parent-fraction.2</b>	Parent fraction of the first input curve, the second, etc. These fractions will be multiplied with <b>plasma.1</b> , <b>plasma.2</b> etc. for the calculation of the input curves. Supported units: [1/1], [%].
<b>start</b>	Start time of the PET/SPECT acquisition frames.
<b>end</b>	End time of the PET/SPECT acquisition frames. Note: the units of the tissue TACs are specified after <b>end</b> , eg. <b>end[seconds]</b>



<b>tac.name</b>	Tissue TAC column with the name specified after the <b>tac.</b> tag.
<b>tac.reference</b> <b>tac.cerebellum</b>	Tissue TACs with this label will be used as reference when applying reference tissue models, if <b>REFERENCE_REGION</b> is not specified in the model section.

## File Format and Loading

The information described above can be saved in a tab-delimited text file (**.kmData**) or an **XLS** (Excel 2003, not xlsx) file and loaded with the **Load KM** file entry from the **Kinetic** menu. Alternatively, the file be dragged and dropped onto the PKIN window or the PMOD ToolBox.

## Data Import from VOIs

An important usage of the VOI analysis is the generation of time-activity curves (TAC) for subsequent kinetic modeling. This can easily be achieved in PVIEW by the following steps

### 1. Definition of the VOIs

The image data is loaded as a dynamic series with the *correct acquisition times* and the *correct input units*. This is important, because otherwise the acquisition start/end times in kinetic modeling will be wrong, and the TACs may be different in magnitude with respect to the blood data. Such problems result in erroneous model parameters.

In dynamic image series there is generally not enough anatomical information to delineate VOIs. Often, averaging of a subset of the acquisition frames resolves the problem. The VOIs are then delineated in the summed images, transferred to the dynamic images, and optionally saved to a file.

### 2. TAC generation

Switch the tool to the dynamic study, and activate the button

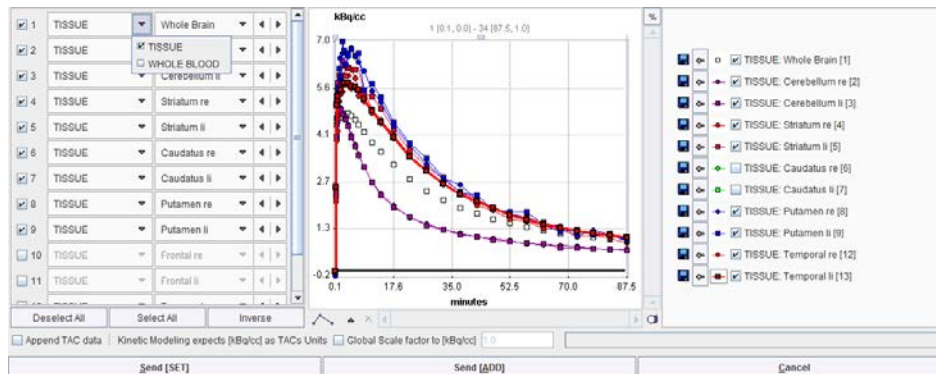


A dialog window appears which is organized in three panels:

- 1) The left area allows defining the proper type (TISSUE, WHOLE BLOOD) of the calculated TACs and selecting the regions to be sent to the **Kinetic modeling** tool.


The central area shows the TACs.

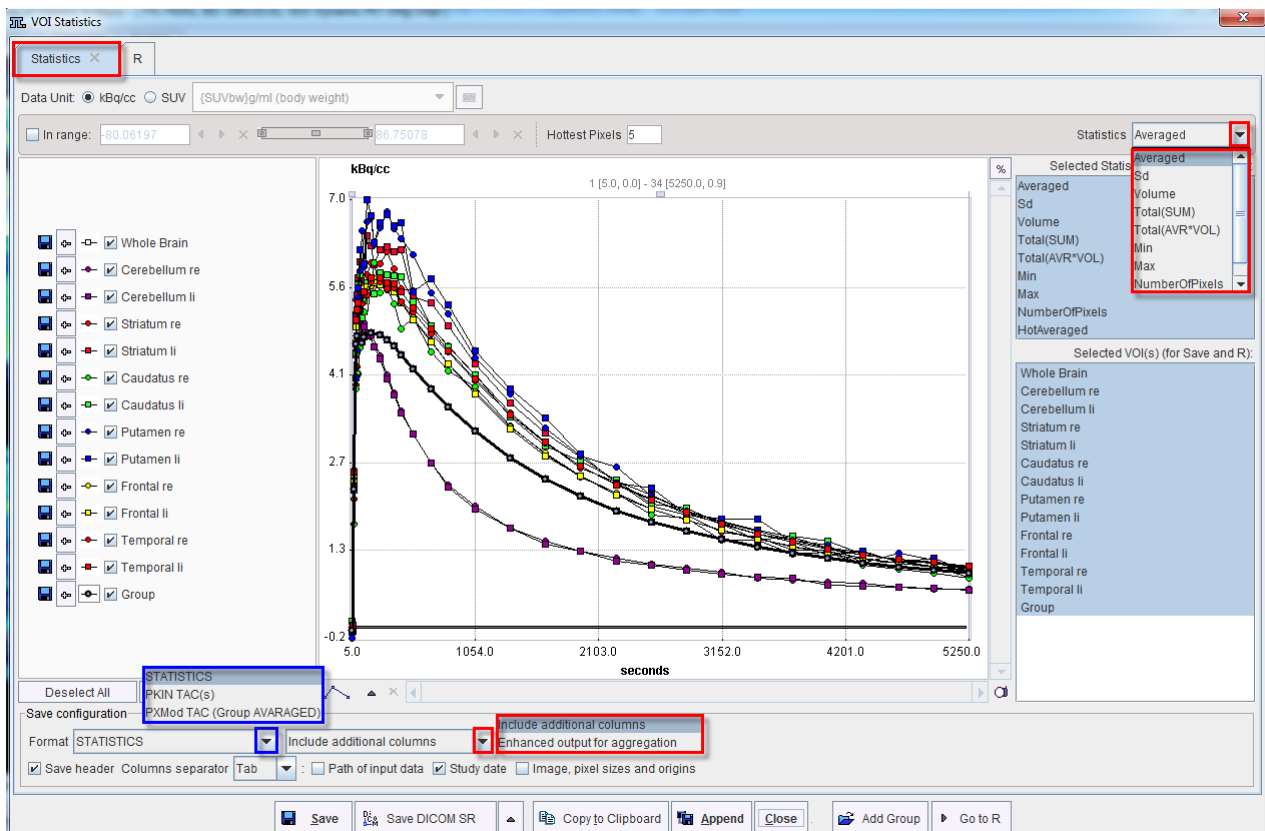
The right area controls the displayed TACs. Additionally, the +- buttons allow for simple arithmetics operations with the curve values before transfer to PKIN.



The **Send** buttons initiate the transfer of the activity curve data to the PKIN tool. Selecting **Send[SET]** transfers the TACs to the currently selected tab in an open PKIN tool. With the **Append TAC Data** box checked, the curves are appended as new regions to the existing data, otherwise the current data is over-written. **Send[ADD]** first creates a new tab in PKIN, to which the data is added. If PKIN is not running, the tool is first started and the data added.

Both the average value and the standard deviation within the VOIs are transferred, as well as patient and study related information. The standard deviation may be used for weighted fits in PKIN.

If the PKIN option is not available, the TACs of a dynamic series can also be obtained with the regular statistics button , resulting in the dialog window illustrated below.



The **In range** box allows restricting the TACs statistics to the subset of pixels with values in a specific range.

Note the radio box in the upper section which allows switching between **kBq/cc** and the **SUV**, if all required information is available. The **Statistic** type selection list allows choosing the type of statistics to be displayed in the graphic area.

The statistics numbers selected in the **Selected Statistic(s) [For Save]** area of the VOIs selected in **Selected VOI(s) [For Save]** can be saved into a text file with **Save**, appended to an existing one using **Append**, copied to the **Clipboard** or sent directly to PMOD R interface with the **Go to R** button. The statistic results can be saved as a Dicom structured report with the **Save Dicom SR** option or exported as a structured report with the **Export Dicom SR**.

There are different statistic formats available for saving procedure:

- ▶ **STATISTICS** saves all selected information. The **Include additional columns** or the **Enhanced output for aggregation** selections are available for the **Statistic** format. We recommend using the **Enhanced output for aggregation** option for aggregation and further statistical analysis in R interface.
- ▶ **PKIN TAC(s)** saves the acquisition times and regional averages in a text file which can directly be loaded with the **Load Time Activity Curve** entry of the **PKIN Menu**.
- ▶ **PXMod TAC (Group AVERAGED)** saves the average of all grouped VOIs in a two-column text file suitable for usage with PXMOD.

## Data Saving

Data can be saved in various formats using the following **Kinetic** menu entries:

<b>Save KM File</b>	Saves all data and the configuration in a comprehensive text file with suffix <b>.km</b> . Loading a <b>.km</b> file restores the state of the previous processing with the exception of the display settings.
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Note that the *model fitting history* (on page 68) is also saved/retrieved.

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<b>Save KM Parameters File</b>	Generates and saves a summary of the model parameters in all regions. Result is a tab-delimited text file with extension <b>.kinPar</b> , which is readable with any text editor and with numerical programs such as Excel. There is a <b>Save</b> and an <b>Append</b> sub-menu, the latter for combining results of several studies in a single file.
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<b>Save all Time Activity Curves</b>	Saves just the time-activity data in a multi-column text file. This option may be helpful to export TAC data for visualization in a different tool.
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Another useful application of **Save/Load Time Activity Curve** is to append a TAC from a different tissue: first save the current TACs, add the TAC of a additional TAC as a new column in Excel, then load the **.tac** file again.

**Save all Model Curves** Exports the tissue model curves of all regions into a text file. Note that these curves are not interpolated between the frame mid-times. To obtain smoother curves please use the **Create Synthetic KM Study** menu item.

---

**Note:** Units of the saved data are always [kBq/cc] and [seconds].

---

## Curve Layout

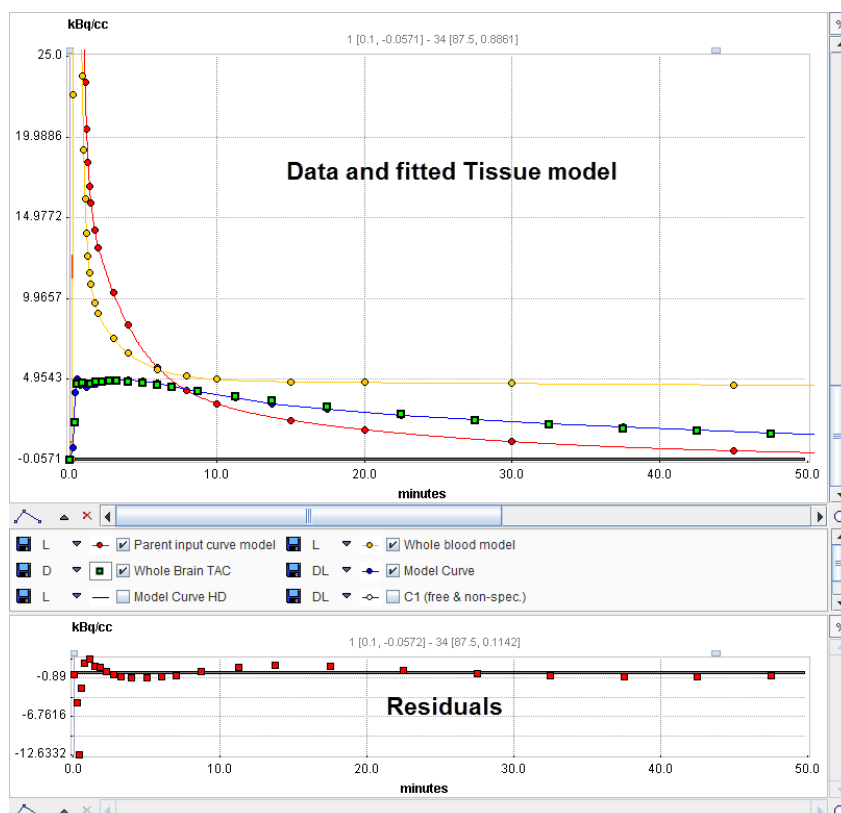
The curve display supports the visualization of the different loaded measurement as well as the calculated model curves. The available curves depends on the selected tab.

### Curves with selected Tissue Tab

After data loading the time-activity informations of the current region are shown in the curve display in a default layout. The large area contains

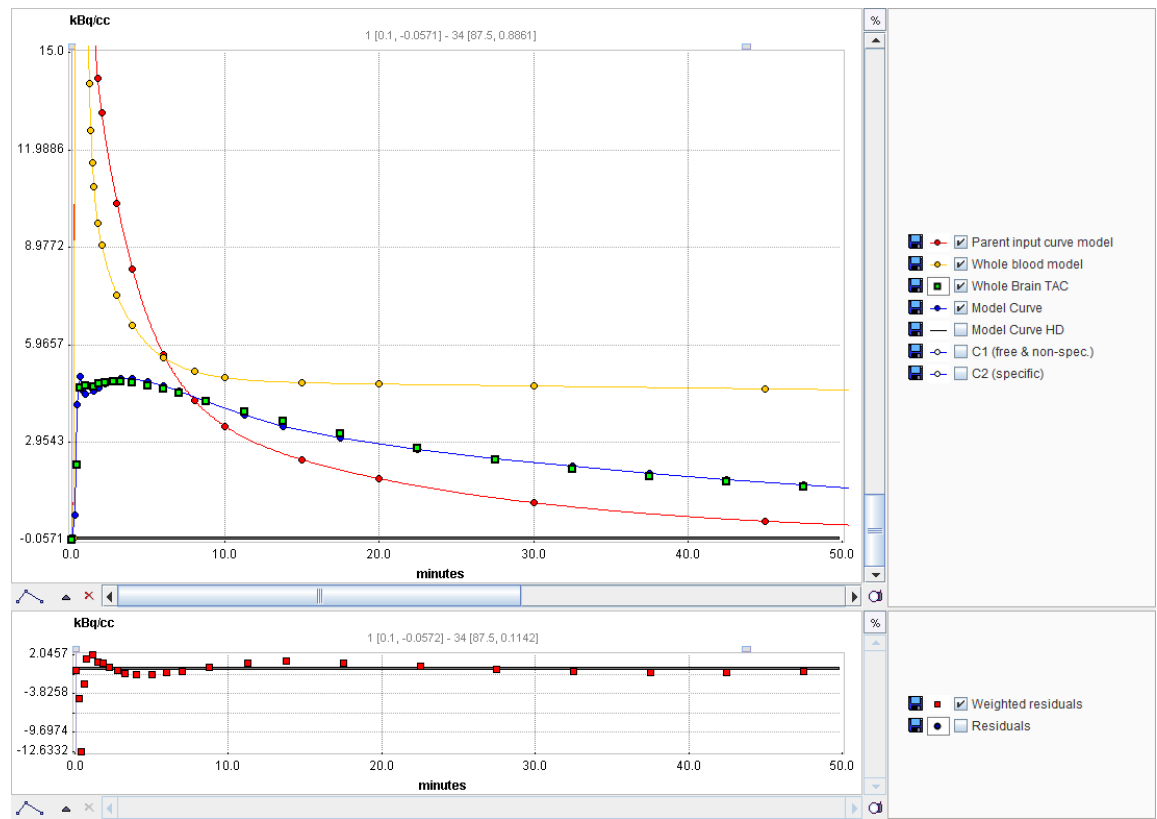
- ▶ the green **TAC** values representing the tissue TAC of the selected **region**, in the example the **Whole Brain TAC**;
- ▶ the red **Parent input curve model** which is used as the input curve for the model calculations;
- ▶ the yellow **Whole blood model** used for blood spillover corrections;
- ▶ and the blue **Model Curve** which results from evaluating the current model configuration with the input curve.

Per default, only the **TAC** and the **Model Curve** are enabled for display by the check box in the control area. Depending on the context, there are additional curves available for display but hidden(box not checked). In the example below the compartment concentrations  $C_1$  and  $C_2$  of the 2-Tissue compartment model as well as a finely resolved **Model Curve HD** could also be shown by checking their boxes.



The smaller curve display at the bottom visualizes the difference between the measured TAC and the model curve. Note that for screens with limited height it may be convenient

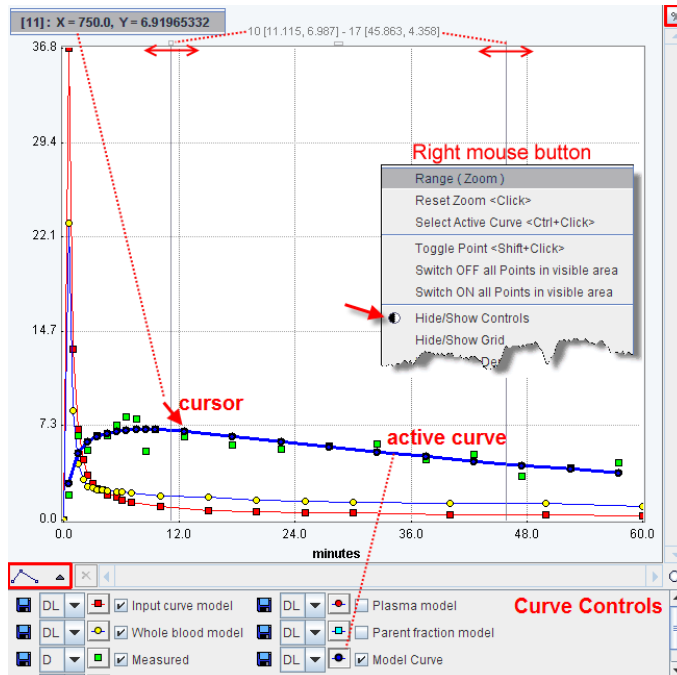
switching the *configuration* (on page 26) to the layout with the curve controls besides the curves as illustrated below.



The general manipulations available in the PMOD curve display are explained in the next section.

## General Curve Display Functionality

A common curve display object is used in all PMOD tools. It consists of a curve area and a controls area underneath.



In some contexts the control area may initially be hidden. The context menu can be used to show it

### Curve Area

The curve area shows the curves which are enabled for display. There is always an *active curve*, which is shown in bold. A curve can be made active by holding down the CTRL key and clicking at one of its points, or by pushing its button in the controls area as illustrated with the **Model Curve** above.

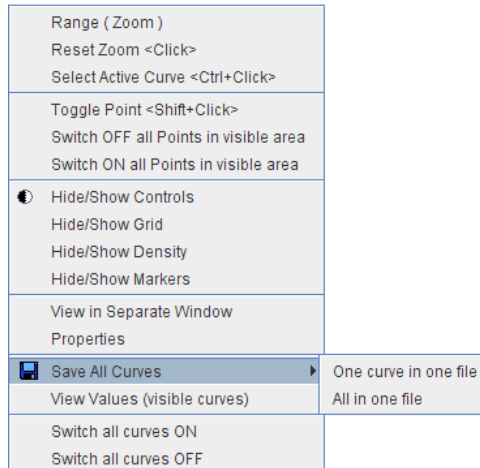
The definition of the active curve is relevant for the tools which interrogate the curve values:

- ▶ There are two small handles at the top of the curve area: a little rectangle to the left, and a line to the right. These are handles which can be moved left/right using the mouse, and the gray vertical lines move with them. The values at the top center of the curve area represent the interpolated (x/y) values of the active curve at the location of the handles. To get the measurements of a different curve just CTRL+Click at that curve to get the values updated.
- ▶ Only in some curve displays: When the cursor is brought close to a point of the active curve, its x/y value pair is shown at the upper left of the curve area.

To *zoom* into an area of the curve just click the left mouse button to the corner of the area of interest and drag a rectangle. After releasing the mouse button the display is zoomed into the defined rectangle. An alternative is to define the axes **Range** in the context menu. A single mouse click into the curve area is sufficient to reset the zoom.

## Context Menu

By clicking the right mouse button into the curve area a context menu with some additional options can be opened.



The functions are:

### Range (Zoom)

Set the range of the x- and y-axis by entering a numeric value.

18.22485	
-1.65517	
[seconds]: 0.0 3944.38	
<input checked="" type="checkbox"/> Fix as curves display range	

If the box is checked, the range is maintained during all manipulations. Otherwise, a single click resets the range to the default.

### Reset Zoom

To reset the curve range to the default full range. It is grayed if the display is not zoomed or the range is fixed.

Mouse operation: single Click into curve area.

### Select Active Curve

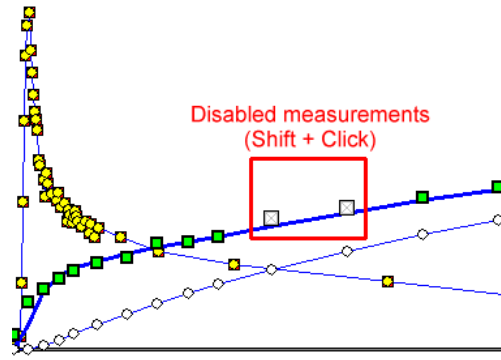
Selects the curve nearest to the point clicked with the right mouse button to open the context menu.

Mouse operation: CTRL+Click at a curve.

### Toggle point


Disable a measurement of the active curve. This is reflected by setting the symbol to gray.





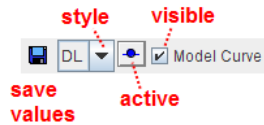
In the context of model fitting measurements marked in gray are regarded as outliers and not considered when evaluating the cost function.

Mouse operation: select the relevant curve to active, then SHIFT+Click at measurement.

<b>Switch ON/OFF all points in visible area</b>	In combination with zooming this option allows to quickly disable/enable contiguous points of the active curve.
 <b>Hide/Show Controls</b>	Allows hiding the controls if the curve display area is small, and to show them again.
<b>Hide/Show Grid</b>	Controls the display of the grid lines.
<b>Hide/Show Density</b>	Reflects the density (coded distance) of points in the graph as a colored map. More points close together produce a "higher" color.
<b>Hide/Show Markers</b>	Controls whether the measurement markers are shown.
<b>View in Separate Window</b>	With this option, the curve display can be opened in a separate, large window to closely examine the plot.
<b>Properties</b>	With this entry a configuration dialog is opened for setting the annotation <b>Font size</b> , and for enabling curve <b>Antialiasing</b> (smooth curve appearance).
<b>Save All Curves</b>	Allows saving the numeric data of all curves in a single or separate text files.
<b>View values (visible curves)</b>	Opens a dialog window which shows the numeric values of all visible curves in a dialog window. The window contents can be copied to the <b>Clipboard</b> and pasted to a different application.
<b>Save All Curves ON/OFF</b>	To quickly change the visibility of all curves. When switching all off, the active curve is still shown.

## Curve Control Area

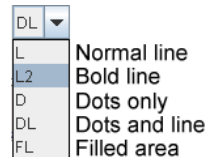
The control area lists the curves which are available for display. There are several elements to modify the curve appearance:



**Show/Hide** To show/hide a curve check/uncheck the *visible* box.

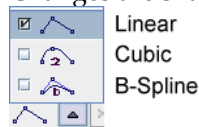
**Active curve** To set a curve to active click at the *active* button, or directly CTRL+Click on the curve itself. The line/symbols get bold.

**Style** The list selection can be used to change the style of a curve:



## Further useful interface elements:

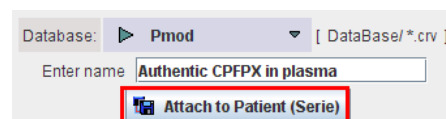
**Connections** Changes the shape of the lines defined by the measurements:



Note that calculations are not based on the display representation of a curve.

✕ Saves the numeric data of a curve as a text file with two columns. These files obtain a **.crv** suffix and can easily be opened in Excel or a text editor.

When working with a database the curve can be attached to a particular image using the **Attach to Patient (Serie)** in the appearing dialog.



% If this button is enabled, each curve is normalized to its own maximum and shown as percent values. This mode is helpful for comparing shapes when the dynamic range of the curves is very different.

🖨 Creates a capture of the curve area. The captured image can be saved as a **JPEG, TIFF or DICOM** file. It can also be copied to the **Clipboard** to paste it into some office application.



## Blood Model Configuration

The tracer activity in blood is normally only sampled at a few time points during the acquisition. However, when calculating the operational equations (eg. by numerically integrating a system of differential equations), the blood activities must be available at any arbitrary time point during the acquisition period. This means that the blood curves must be interpolated according to some underlying model, hence called a "blood model". The interpolation models available for the different blood curves are described in the corresponding model reference section.

### Input Curve (AIF)

PKIN supports four types of blood data which together allow computing the *input curve* (on page 12).

- 1) The tracer activity concentration in whole blood. This information is used for the spillover term in the compartment models as well as in combination with the plasma fraction.

The total activity concentration of tracer in the plasma part of the blood.

The plasma fraction (ratio of tracer in plasma to tracer in whole blood). The plasma fraction is multiplied with the whole blood concentration to calculate the total concentration of tracer in plasma.

The parent fraction (ratio of unchanged tracer to total tracer in plasma). The parent fraction is multiplied with the total plasma concentration to calculate the input curve.

Note that the plasma options 2. and 3. above are excluding each other. If plasma activity has been loaded, the plasma fraction option cannot be used. On the other hand, if the plasma fraction has been loaded, the plasma activity option cannot be used.

The different possibilities to calculate the input curve work as follows:

- ▶ As soon as a whole blood curve is loaded and no other information is available, the plasma mode is set to "fractional" and a **Fix** model is applied with a factor of 1. This means in practice that the plasma activity is set equal to the whole blood activity.
- ▶ As soon as a plasma activity curve is loaded any plasma fraction data is discarded, and the plasma mode set to "activity". The activity interpolation mode defaults to **Lin. Interpolation**. The parent fraction is not changed.
- ▶ As soon as a plasma fraction curve is loaded any plasma activity data is discarded, and the plasma mode set to "fractional". The fraction interpolation model defaults to **Lin. Interpolation**. The parent fraction is not changed.
- ▶ A parent fraction equal to 1 is applied as long as no parent fraction curve is loaded, or no analytical parent fraction model selected.

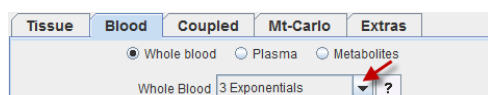
The different blood-related informations are readily available for inspection in the curve area of the **Blood** tab as illustrated in the sections below.

## Whole Blood and Plasma Activity

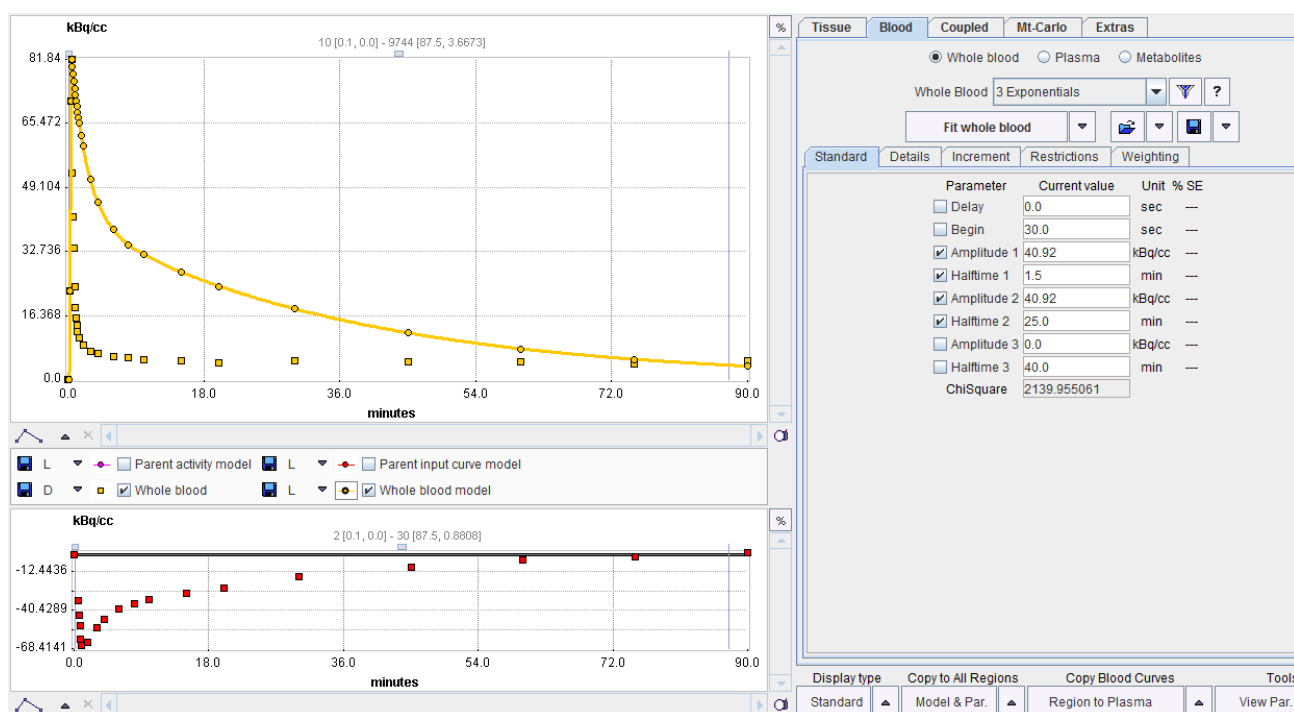
The interpolation of the activity curves of whole blood and plasma are performed using the same models.

### Whole Blood Model Configuration

It is assumed that the time-course of the tracer activity in whole blood has been loaded with **Kinetic/Load Whole Blood**. To configure the interpolation model of whole blood please select the **Blood** tab, and set the **Whole blood** radio button. A list of models is available which can be shown with the arrow button indicated below.



Default is **Lin. Interpolation**, which just represents linear interpolation between sample times. To replace linear interpolation by a smoother function select an appropriate definition from the list. As soon as a model is selected, the parameters are updated in the **Standard** pane, and a corresponding model curve is shown in the curve window as **Whole blood model**.

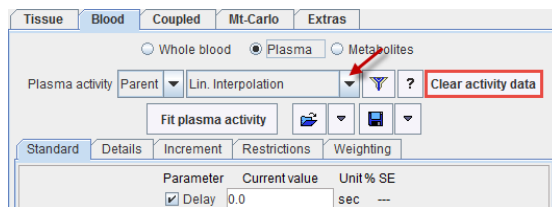


In the example above the model has not yet been fitted so that the distinction between the measurement and the model (sum of exponentials) is clearly visible. The parameters of the blood model can be configured for fitting purposes by enabling the check boxes. Activating the **Fit Whole Blood** button starts a fitting process which adjusts the model parameters such that the interpolation curve comes into optimal agreement with the measurements. Fitting works exactly in the same way as was explained for the tissue models *below* (on page 62).

## Plasma Activity Model Configuration

Similarly to above, it is assumed that the time-course of the tracer activity in the arterial plasma has been loaded with **Kinetic/Load Plasma Activity**. The configuration of the interpolation model works in exactly the same way as for whole blood.

Please select the **Blood** tab, and set the **Plasma** radio button. Note the label **Plasma activity** which indicates that the current working mode uses measured activities, not a derivation from whole blood activity. The **Clear activity data** button is available to change this mode. If it is activated, the loaded plasma activity data is discarded and the mode switched to plasma fraction. In that case the assumption is that an analytical plasma fraction function will be applied to the whole blood activity curve to derive the plasma activity.



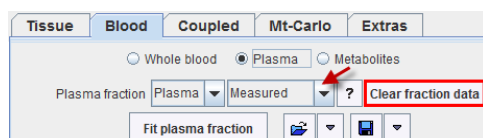
The same list of models is available as for the plasma activity as for whole blood. The available models are described in the reference section *section* (on page 185).

## The Blood Delay Parameter

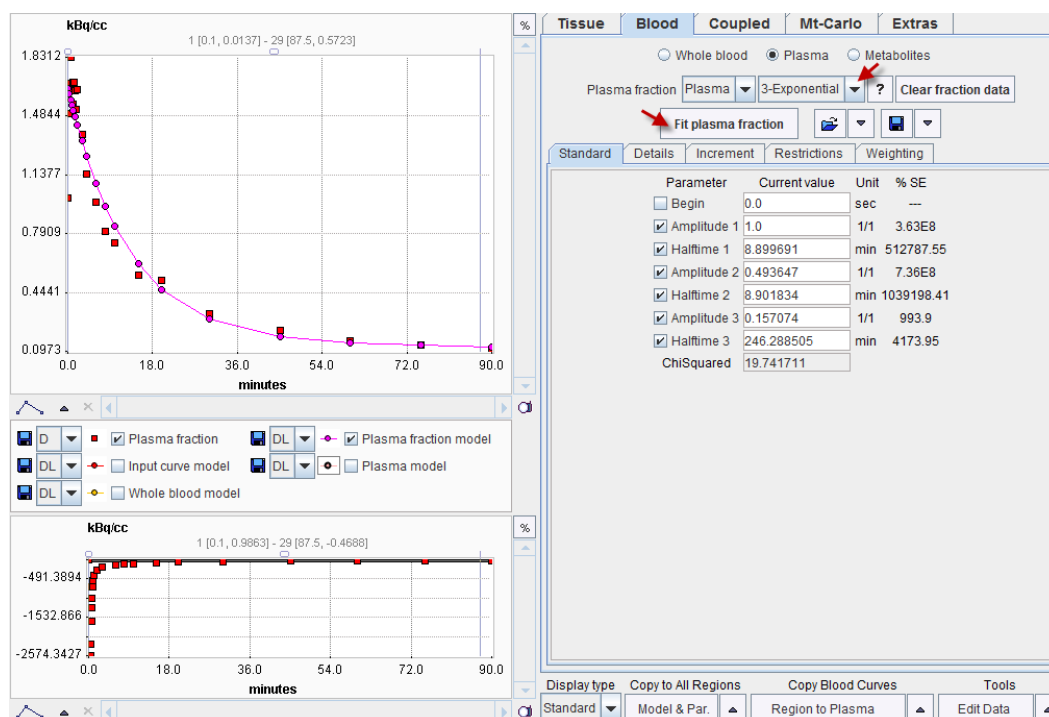
All standard blood models have a **Delay** parameter to correct for a timing error between tissue and blood data. Positive delays represent delayed blood information and hence shift the blood curves to earlier times (to the left). This parameter is only relevant for fitting of the *tissue* model. Therefore, when fitting the *shape* of the blood curve with **Fit plasma** or **Fit whole blood**, it is automatically *disabled*. When *fitting the kinetic tissue model* (on page 77), however, it can be allowed for fitting.

## Plasma/Whole-blood Fraction

For the following description it is assumed that the time-course of the plasma fraction has been loaded with **Kinetic/Load Plasma Fraction**. To configure the interpolation model please select the **Blood** tab, and set the **Plasma** radio button.



Note the label **Plasma fraction** which indicates that the current working mode uses a function to derive plasma activity from whole blood activity. There is a list of models which can be fitted to the plasma fraction data. As soon as a model is selected, its parameters are shown in the **Standard** pane, and a corresponding model curve is displayed in the curve window as **Plasma fraction model**.



The **Clear fraction data** button is available to remove the loaded plasma fraction data. In that case the assumption is that an entirely analytical plasma fraction function will be applied to the whole blood activity curve in order to derive the plasma activity.

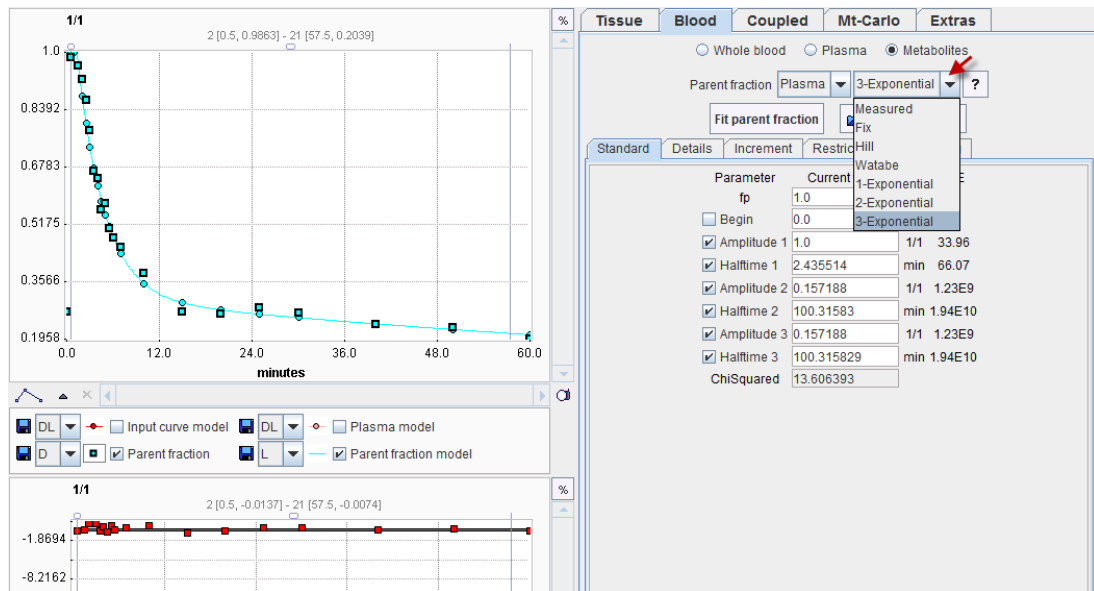
The available models are described in the reference *section* (on page 194), and *fitting* (on page 57) works in the same way as explained for the tissue models.

## Parent/Plasma Fraction

The transformation of the plasma activity curve into the input curve is governed by the configuration on the **Metabolites** tab. Two cases are to be distinguished.

### Measured Parent Fraction Data is available

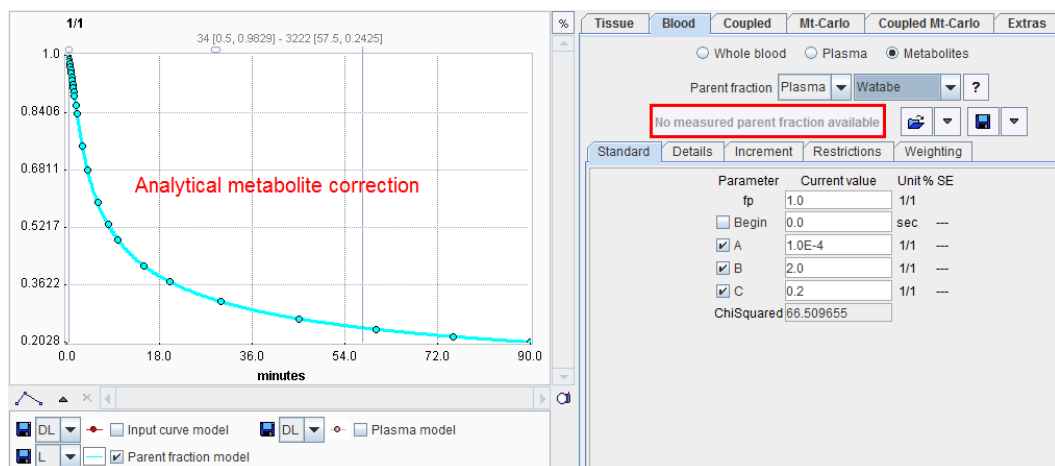
If a parent fraction curve has been loaded, it is shown in the curve area. Initially, the parent fraction model is set to **Lin. Interpolation** so that linear interpolation between the samples is used to calculate intermediate parent fractions. The model selection choice allows to select smoother models, and fit them to the measurements using the **Fit parent fraction** button as illustrated below.



### No Parent Fraction Data available

If no data parent fraction data have been loaded, the parent fraction model is set to **Fix** with a free parent fraction  $f_p=1$ . This is adequate for all tracers which do not require metabolite correction, and for cases when the metabolite correction has already been applied to the plasma data prior to loading them with **Load Plasma Curve**.

In the case of a tracer which shows a characteristic buildup of the metabolites it may be possible to derive an average parent fraction curve and fit a model to it. To apply a metabolite correction the user can then simply select the appropriate model function and enter the demographic parameter values, as illustrated below. To apply a demographic correction, it must be ensured that time zero of plasma data always represents the time of tracer injection. Naturally, no fit will be possible in this situation.



The available models are described in the reference *section* (on page 201), and *fitting* (on page 57) works in the same way as explained for the tissue models.



---

**Note:** The metabolite correction is applied to the plasma curve using the original plasma sampling times. The assumption is that the parent fraction and the plasma curve have a common time scale. Only after the metabolite correction has been applied is the input curve shifted by the blood sampling time delay.

---

## Default Model Configurations and Configuration Sets

All interpolation models allow the customization of the model default parameters, and the saving of specific configuration sets for certain task. The *mechanism* (on page 61) is the same as described for the tissue models.

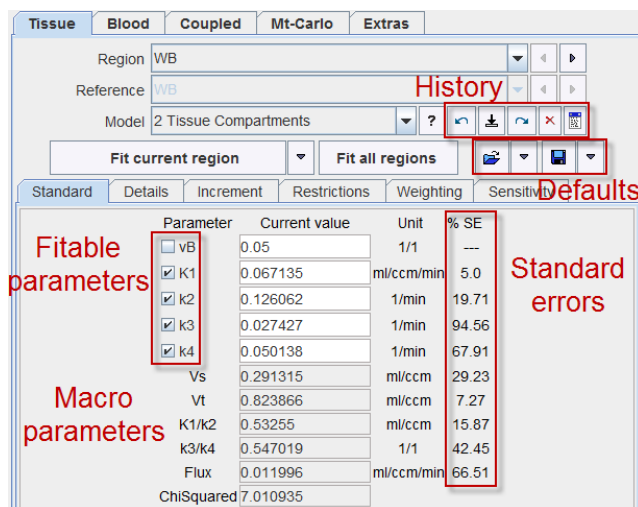
# Fitting of Kinetic Tissue Models

As soon as a kinetic model has been selected a model curve is calculated and shown in the curve display. The model curve represents the activity concentration values PET would measure if the model correctly described the observed physical processes. Fitting assumes that the model configuration is adequate and tries to find the set of model parameters for which the model curve is in optimal agreement with the measurement.

The following sections explain how a model is configured, fitted to data, and assessed for goodness-of-fit.

## Kinetic Model Configuration

The main functionality for the definition and fitting of kinetic tissue models is available on the **Tissue** pane. The upper part contains data and model selections, while the lower part gives access to the model details and fitting results.



The main elements for the model configuration on the **Tissue** pane are:

**Region** When several regional TACs have been defined and imported, the current regional TAC can be set using this list selection. The left and right arrows allow stepping forwards and backwards through the TACs.

After region switching, the display is updated with the configured model and the curves of the selected region.

**Reference** Selection of the regional TAC serving as reference in models which have the input curve replaced by a tissue TAC. Note that **Edit Data** (on page 99) allows creating a TAC averaged from several loaded TACs, which may be useful as a reference.

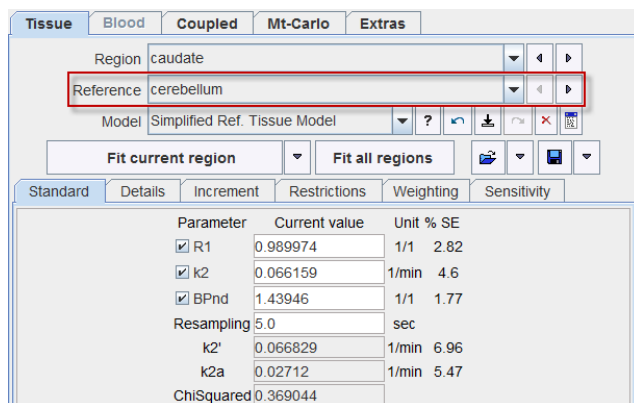
**Model** The PKIN tool includes a comprehensive list of standard kinetic models and some auxiliary calculations. This list allows selecting among all currently configured models. As soon as the model is changed, the pane in the lower section is updated to show the parameters of this model.

Note that the models in the list can be rearranged, added or deleted using the **PKIN Models** tab after selecting **Config** from the PMOD ToolBox. Furthermore, the list can be focused on the current analysis by *filtering* (on page 62).

? Button for showing a quick information about the current model.

## Reference Models

When a reference **Model** is selected which uses the activity in a reference region as an implicit input curve, the blood-related items become disabled and the **Reference** region selection becomes active.



PKIN will pre-select a region containing "Cerebellum" or "Reference" in the name as the reference. If the selection is not adequate, please select the proper region from the **Reference** list.

## Model Parameters

The **Standard** sub-tab illustrated above shows the configuration of the currently selected model. It encompasses different types of parameters:

- » Input parameters which are needed to specify information used for model calculations.
- » Fitable parameters which require a starting value and show the updated results after model fitting. A parameter is only optimized if its fit box is checked, otherwise it remains fixed.
- » Macroparameters are calculated from model parameters in the upper part to instantly provide composite information such as the distribution volume or the flux.

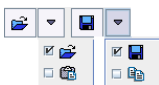
The fitable and derived parameters have four columns of information:


- The **Parameter** name. In order to see a brief explanation of the parameter place the mouse pointer over the name. The explanation text is then displayed in the status line at the bottom.
- The **Current value**, which can be edited, and which is updated as the result of a fit.
- The **Unit** of the parameter value.
- The standard error %SE of the parameter expressed as a percentage of the parameter value itself. It is only available, if a fit has been performed using the **Marquardt-Levenberg** method.


## Default Model Configurations and Configuration Sets

### Model Default Configuration

Each model has default values. Initially, they are factory settings, but they can be re-defined by the user if he wants to establish a default configuration which is more adequate from him. There are four buttons next to **Fit all regions** for saving and retrieving the model configurations.




 Save the current parameter configuration as the new default for the particular model type. Included are the values, the fit flags, and the restrictions of all parameters.


 Resets the configuration of the current model to the saved defaults.

Note that the configuration defaults are managed on the basis of the PMOD login. Consequently, different logins could be prepared for processing types of data requiring different initial parameter values.

### Complete Model Configuration Set

A model configuration can also be saved as an explicit definition file (**.kmModel**). It includes the tissue model as well as all blood models used. Included are the initial parameters, the fit flags, the restrictions and the residual weighting scheme (except for the prescribed weighting with the explicit weighting factors).

 Save the current model and parameter configuration in a file.

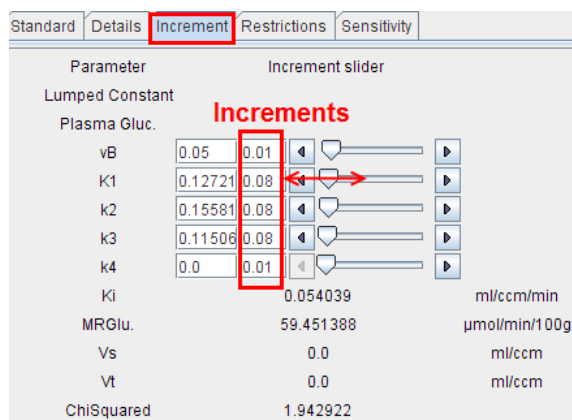
 Retrieve a model configuration.

The configuration files make it simple to establish defaults for particular data processing tasks: just load the appropriate **.kmModel** file and set it as the default. Another application is *batch processing* (on page 90). There, multiple models can be fitted to the data by simply selecting the corresponding **.kmModel** files.

### Visualization of Parameter Effect

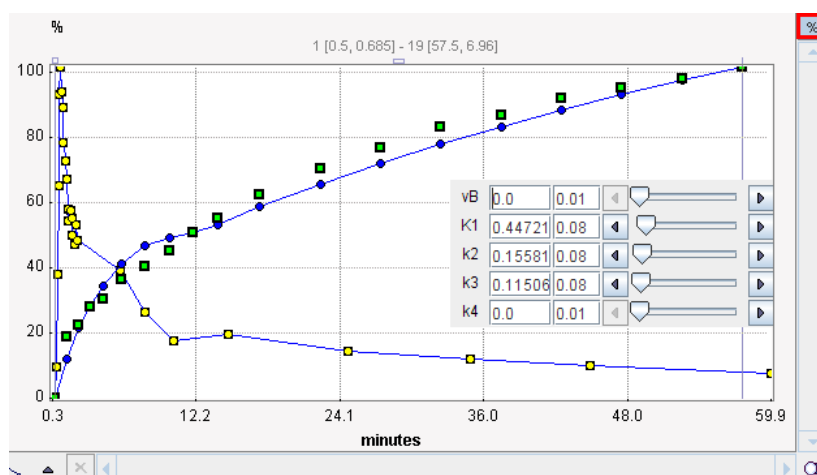
The **Increment** sub-tab is mainly used for visualization purposes. For each fitable parameter the current value is shown next to an increment value and an increment slider. When the

arrow buttons are selected, the parameter value is incremented or decremented accordingly. The slider allows to increment/decrement a parameter smoothly. The minimal and maximal values for the slider as well as the initial increment are obtained from the values on the **Restrictions** tab.



After every parameter change the curve display is updated, so that the user can observe its effect on the global uptake (model curve) and on the uptake in the different compartments ( $C_1(t)$ ,  $C_2(t)$ ).

For instance, as shown below, it can be nicely demonstrated that a change in  $K_1$  simply scales the model curve, but does not change the curve shape: Set the curve display to relative scaling by selecting the % in the upper right and then change  $K_1$ . Nothing changes in the % display, provided that the blood volume fraction is set to 0. Any modification of the other parameters changes the curve shapes notably.



## Parameter Restriction

The **Restrictions** sub-tab is relevant for fitting. For each fitable parameter there is a box for enabling the restriction which is defined by the **Lower bound** and the **Upper bound** value. The purpose of parameter restrictions is to ensure that the resulting parameters remain within a reasonable physiologic range. Without restrictions, there is a higher chance to end up with a meaningless solution. Note that these bounds are also used for the sliders on the **Increment** pane.

The **Search** value is used as an initial step-size for iterative fitting. It is recommended to set this value to about the same size as the parameter value itself, as it is automatically decreased if needed. This approach is not successful if the value is very small from the beginning.

Parameter	Lower bound	Upper bound	Search
Lumped Constant			
Plasma Gluc.			
<input checked="" type="checkbox"/> vB	0.0	1.0	0.01
<input checked="" type="checkbox"/> K1	0.0	0.8	0.1
<input checked="" type="checkbox"/> k2	0.0	0.5	0.1
<input checked="" type="checkbox"/> k3	0.0	0.5	0.1
<input checked="" type="checkbox"/> k4	0.0	0.5	0.1

## Conversion of Model Parameters with Model Switching

There are certain relations between the parameters of *compartment models* (on page 7) with different complexity. Accordingly, PKIN can be configured to convert the rate constants when switching between compartment models. This mode is mainly useful when developing from a simple model towards a more complex one. Conversion is enabled with the **Model conversion** check in the **Extras** pane.

Tissue	Blood	Coupled	Mt-Carlo	Extras
<input type="checkbox"/> Parameters initialization <input type="checkbox"/> Model conversion <input type="checkbox"/> Fit blood <input type="checkbox"/> Random fits <input type="text" value="20"/> <input type="checkbox"/> Data shortening to [min] <input type="text" value="20.0"/> Fitting method <input type="text" value="Marquardt"/> Max. iterations <input type="text" value="200.0"/>				

Behavior if the user switches the model between different compartment models:

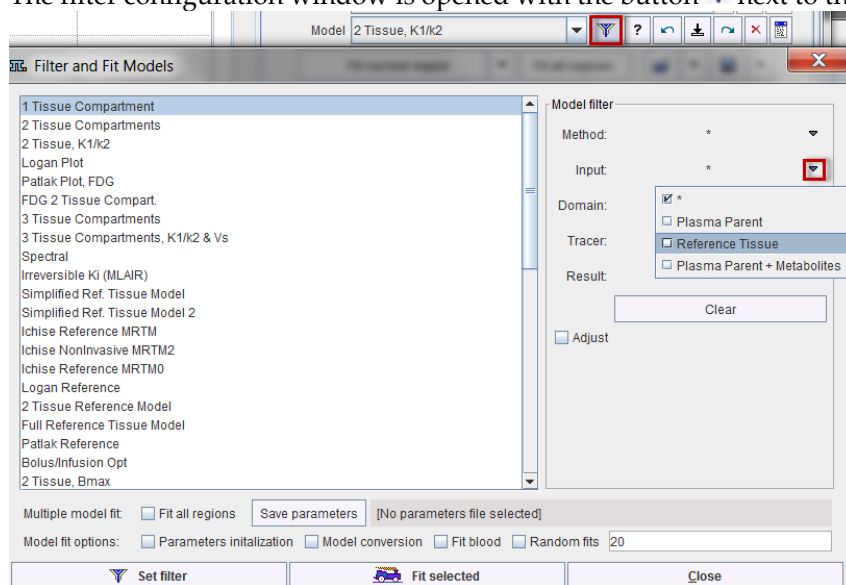
- ▶ Model conversion disabled: the first time a model is selected, its default values are used, and thereafter the most recent set of parameter values of that particular model. This behavior works on a per-region bases.
- ▶ Model conversion enabled: Each time, the user switches between the models, the parameters are converted. If switching is between models with the same number of compartments, the conversion is exact. If the number of compartments is decreased, the lumped parameters of the reduced model are calculated. If the number of compartments is increased, the same values are used for corresponding parameters, and the additional parameters set to the default values. The consequence of this functionality is that when switching the model from 1-tissue to 2-tissue and then back, the  $K_1$  and  $k_2$  values will be different due to this conversion process.

The conversion is also meaningful for reference tissue models which rely on  $k_2'$  as an input parameter. The conversion will try to maintain  $k_2'$  when switching between reference models with explicit  $k_2'$ .

## Model Filtering

The *basic model configuration* (on page 24) should include all models which may be relevant for a user. Consequently, the model list in the PKIN user interface can still be quite long. Model filtering can be used to quickly tailor it to the current task.

The filter configuration window is opened with the button  next to the model list.



The **Model filter** area has five classifiers which can be enabled

- |               |                                                                                                                                                                                        |
|---------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Method</b> | Selections: <b>Classical PET</b> , <b>Compartment Model</b> , <b>Graphical Plot</b> , <b>Spectral</b> , <b>Multilinear</b> , <b>Utility</b> .                                          |
| <b>Input</b>  | Selections: <b>Plasma Parent</b> for blood-based models, <b>Reference Tissue</b> for reference models, <b>Plasma Parent + Metabolites</b> for models with metabolites entering tissue. |
| <b>Domain</b> | Selections: <b>Tissue Perfusion</b> , <b>Receptor Binding</b> , <b>Heart Perfusion</b> .                                                                                               |
| <b>Tracer</b> | Selections: <b>Reversible</b> , <b>Trapping</b> , <b>FDG</b> , <b>NH3</b> , <b>H2O</b> , <b>Acetate</b> , <b>Rb</b> .                                                                  |
| <b>Result</b> | Selections: <b>Vt</b> , <b>BPnd</b> , <b>Flux</b> , <b>Bmax</b> .                                                                                                                      |

After selecting one or more filter classifiers the model list in the window is reduced accordingly.

The **Adjust** flag implements a prior condition on the selected models. Currently, only the **Trapping** has an impact. It causes setting  $k_4=0$  in the compartment models. Use the **Set filter** button to establish the current list for PKIN and close the window.

## Kinetic Model Fitting

The main elements on the **Tissue** pane for model fitting are:

<b>Fit current region</b>	Starts the process which fits the model curve to the measurements of the selected regional TAC. For compartment models the fit is an iterative optimization, while for some other models only a single calculation may be sufficient.
<b>Fit all regions</b>	Performs fitting for all TACs of the study using their current regional model definitions.

## Iterative Fitting Principle

Except for linear and multi-linear regression, minimization of the *Chi square* (on page 15) expression requires an iterative approach. The procedure starts with a set of initial parameter values, generates the corresponding model curve, calculates the Chi square criterion, and adjusts the model parameters such that Chi square is expected to be reduced. The cycle

- » chi square calculation
- » parameter adjustment

is then repeated until a further reduction of Chi square is not any more possible. The final set of parameters upon termination is regarded as the best-fit result. It is not analytically exact and may depend on the initial parameters, on the optimization procedure, and on the termination criterion of the iterations. Fitting is started using the **Fit current region** button.

## Fitting Options

The **Extras** pane contains some options which have an impact on model fitting.

<b>Parameters initialization</b>	<p>This configuration is only related to the compartment models and determines which initial values are used for the iterative fitting:</p> <ul style="list-style-type: none"> <li>» Disabled: the user-defined parameters are used.</li> <li>» Enabled: an automatic procedure determines initial parameters which should be reasonably close to the solution. This is done by the <i>linear least squares method</i> (on page 114). Note that the initial values are not shown to the user. However, to see the values, please switch to the linear least squares method and fit.</li> </ul>
<b>Model Conversion</b>	Behavior of the parameters when <i>switching between models</i> (on page 61).
<b>Fit blood</b>	If this box it is checked on, <i>all</i> parameters of the <b>Input</b> curves which have



the fit box checked are also included into the set of fitted parameters. This feature has two uses:

- ▶▶ Fitting the relative time delay between the tissue and the blood data simultaneously with the kinetic rate constants. However, it is easier to directly use the *dedicated button* (on page 77).
- ▶▶ (Rarely) Fitting a shape parameter such as the half-time of a metabolite build-up together with the rate constants as explained *below* (on page 92).

#### Random Fits

This configuration is only related to models which require iterative fitting:

- ▶▶ Disabled: Only a single fit is performed and the result parameters returned.
- ▶▶ Enabled: Multiple fits are performed, and the result parameters of the fit with minimal chi squared returned. The fits are started with randomized sets of initial parameters using a uniform distribution with  $\pm 100\%$  range.  
Note: If **Parameters initialization** is enabled, randomization uses the automatically determined parameters for the random number generation.
- ▶▶ In case randomized fitting with a number n of trials is enabled, the label of the fitting buttons is adjusted correspondingly.

---

**Note:** The purpose of randomized fitting is to avoid local minima, but there is still a chance that the returned solution may contain unphysiologic parameter combinations.

---

#### Data shortening to

Enables a sequence of fits with a successively shorter data segment to study the *parameter sensitivity to acquisition duration* (on page 86).

#### Fitting method

Selection of the numerical optimization method applied for iterative fitting:

- ▶▶ **Marquardt:** Marquardt-Levenberg algorithm [59]. Usually works faster and has the additional advantage that it accumulates information about the covariance matrix. Therefore, an estimate of the standard error can be calculated for the fitted parameters. They are shown as %SE (coefficient of variation), ie. as a percentage of the parameter value.
- ▶▶ **Powell:** Conjugate direction set method [59].

The iterative optimizations terminate, when no substantial improvement can be found any more, or when the **Max iterations** restriction is hit.

---

**Note:** Several models (linear and multi-linear regression) are not fitted iteratively because they have closed form solutions. In these cases the **Fitting method** settings are not relevant.

---

**Max. iterations** Maximal number of iterations which are allowed in the iterative **Marquardt and Powell** fitting.

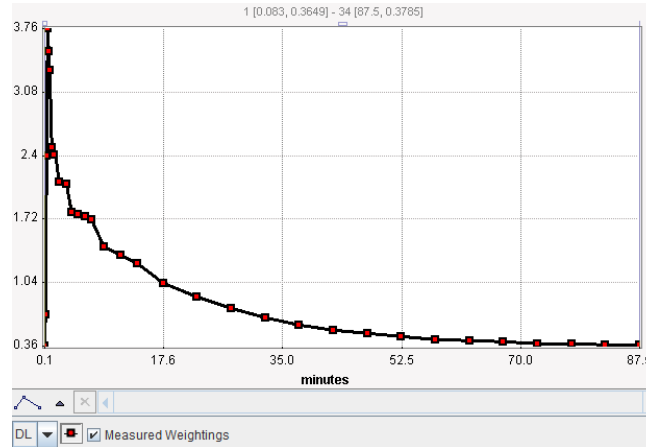
## Residual Weighting

**Weighting** (on page 15) is used to increase or decrease the influence of certain data samples on the fitting. For instance, initial samples with short acquisition duration and little uptake may be considered less reliable than the samples with longer durations in the presence of high uptake.

In PKIN, several different weight definitions are supported. They can be specified on the **Weighting** pane using the list selection with the three entries **Constant weighting**, **Prescribed weighting** and **Calculated weighting**. The contents of the different selections are illustrated in parallel below.

**Constant weighting** With constant weighting the same weight is applied to all residuals. Its size can be adjusted by the **Scale** factor and the radio button setting. In the case of **TAC mean**  $\sigma_i$  is obtained as the average of all values in the TAC multiplied by the **Scale** factor. Similarly, with **TAC max**  $\sigma_i$  is obtained as the maximal TAC value multiplied by the **Scale** value. With **All weights = 1** the weights are all set equal to 1 independent of the **Scale** ( $\sigma_i = \sigma_i^2 = w_i = 1$ ).

**Prescribed weighting** The default behavior of this method is to use the standard deviation of the pixel values in the VOI for calculating the TAC for weighting. This information is automatically available if the TACs have been transferred from the PMOD viewing tool to PKIN, and can be visualized using the button **Show measured stdv** as illustrated below.



This definition can be overwritten by user-defined weights which are to be loaded with the **Load stdv from file** button. Please see below how the file has to be formatted.

### Calculated weighting

With this selection the variance is calculated for each TAC value from an equation which may take into account the measured uptake, the radioactive decay, and the duration of the acquisition. The full equation is as follows:

$$\sigma_i^2 = \alpha \frac{C_{PET}(t_i)}{\Delta t_i e^{-\lambda t_i}}, \quad \lambda = \frac{\ln 2}{T_{1/2}}$$

It includes a **Scale** factor  $\alpha$ , the decay-corrected **Uptake**  $C_{PET}(t_i)$  at frame mid-time  $t_i$ , the frame **Duration**  $\Delta t_i$ , and the **Decay** constant  $\lambda$  which is obtained from the half-life  $T_{1/2}$  of the isotope. If the half-life has not yet been defined, it can be set on the **SUV** panel with the *Edit Patient* (on page 100) facility. The settings of the radio buttons have the following effect on the equation above for calculating  $\sigma_i^2$ :

#### » Uptake:

**Not considered:**  $C_{PET}(t_i)$  is replaced by 1.

**Measured value:**  $C_{PET}(t_i)$  is used, corresponding to *Poisson weighting*. The effect is that more emphasis is given on low-uptake values than on high-uptake values. This may be helpful for example if the fit seems to not account enough for the tail of a decreasing function such as the input curve.

**Measured value squared:**  $C_{PET}(t_i)$  is replaced by  $C_{PET}^2(t_i)$ , corresponding to *Relative weighting*. With this option the influence of the uptake is further increased. It should be used with care because by squaring small uptake values ( $\ll 1$ ) very small variances are generated which transform into heavy weighting. As a consequence, a few small uptake values may have too much impact on the fit.

#### » Duration:

**Not considered:**  $\Delta t_i$  is replaced by 1.

**Measured value:** The individual frame durations  $\Delta t_i$  are used, so that longer acquisitions are considered more reliable.

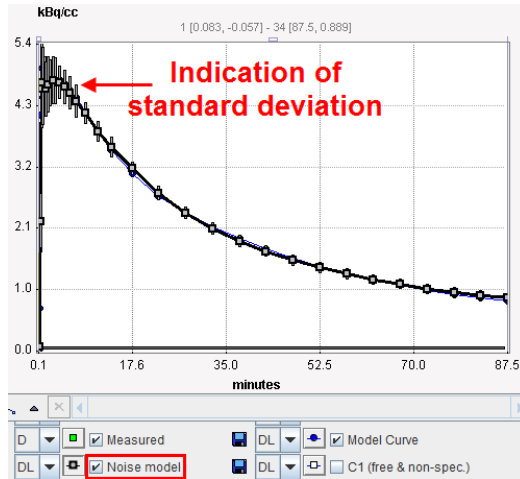
#### » Decay:

**Not considered:**  $e^{-\lambda t_i}$  is replaced by 1.

**Frame mid-time:**  $e^{-\lambda t_i}$  is used, so that earlier acquisitions are considered

more reliable.

Whenever the configuration is changed, the standard deviation  $\sigma_i$  is calculated and plotted as an error bar around the measurements. This gives the user a visual feedback as a help in judging the adequacy of the specified weighting.



### Loading Externally Defined Weights

Some users might want to implement their own residual weighting schemes. To this end they should calculate the standard deviations  $\sigma_i$  for the different acquisition frames from which the weights will be calculated according to

$$w_i = \frac{1}{\sigma_i^2}$$

The  $\sigma_i$  should be prepared in a tab-delimited text file of the following form:

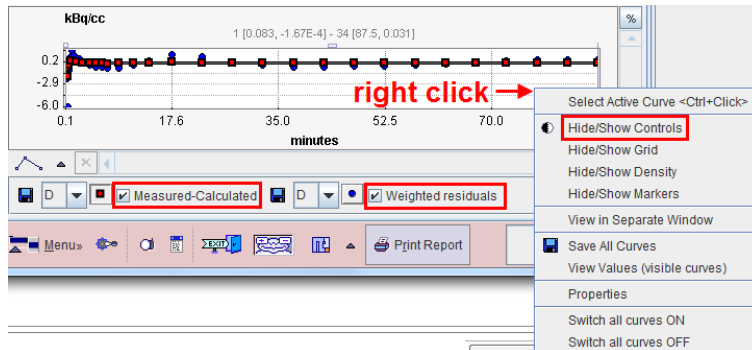
t[seconds]	standard-dev[kBq/cc]
10	0.10300076
30	0.01717448
50	0.01195735
70	0.01010257
etc	...

The header line is required. The first column represents the frame mid-times (although the values are not interpreted), and the second column the standard deviations in appropriate units. The number of entries must be equal to the number of acquisitions, and the columns separated by spaces or tabs. For example, such a file can be prepared in MS Excel and then saved as a text file with tab delineation.

This definition can be loaded by the **Load stdv from file** button available with **Prescribed weighting**.

## Display of the Residuals

The curve area with the residuals per default shows the weighted residuals. However, the raw residuals can also be available. The context menu as illustrated below (right mouse click into the curve area) can be employed for showing/hiding the curve controls, which allow changing the displayed information.



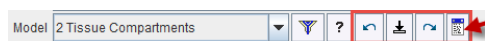
## Use of the Weights in Monte Carlo Simulations





Monte Carlo simulations require the generation of noise which is defined by a distribution type as well as its deviation characteristics. Assuming that the weights specification corresponds to the standard deviation of the measurement noise, the regional weighting specification is used in the Monte Carlo noise generation together with the distribution type. Note that the **Scale** factor has no impact on fitting, but that it is highly relevant in Monte Carlo simulations.

## Model Fitting History

The system maintains a history of model configurations for each region. All operations, which change the model configuration result in a new entry in the history list. The history is saved within a .km file and will be continued after reloading the data.

The history functionality is accessible by the user interface elements right to the model configuration

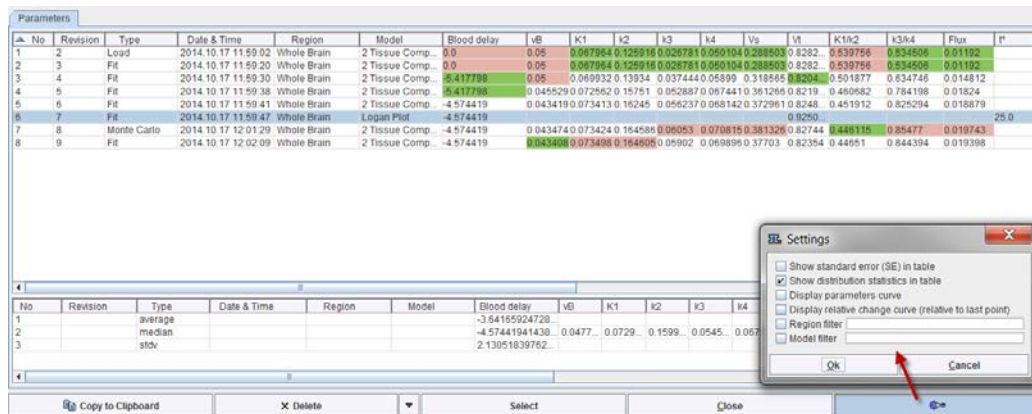


-  Switches to the previous model configuration in the history list.
-  Appends the current model configuration to the list.
-  Switches to the following model configuration in the list.
-  Shows the history window.

**CAUTION:** This history mechanism does not account for changes in the configuration of the blood-related models, except for the blood delay. However, it includes the definition of the residual weighting, the valid points, and the tissue TAC values.

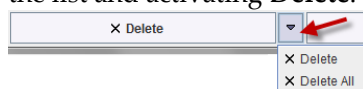
## History Window

The history list is shown in a dialog window as illustrated below.



The upper part of the **Parameters** panel lists the model configurations in rows. The initial columns are descriptive, followed by the actual parameters. Note that the parameters are sorted into appropriate columns even across different model types. For instance, **Vt** resulting from all compartment models as well as the Logan plot appears in the same column so that the values can easily be compared. The user can change the order of the columns by dragging the column headers left or right. The smallest value per column is encoded in green, the largest in red. Columns having a constant value are encoded in blue. The **No** column refers to the chronology of list entry. **Revision** serves for grouping results from a certain task such as the *parameter sensitivity analysis* (on page 86). **Type** indicates by what process the configuration was added. The list can be flexibly sorted by clicking on the column headers.

To restore a model configuration double-click the list entry or mark it in the list and activate **Select**. **Copy to Clipboard** copies the window contents, so that it can be pasted into another application for further analysis. Model configurations can be removed by selecting them in the list and activating **Delete**. The **Clear All** button



allows resetting the history completely. **Close** just closes the window.

## Optional Elements

The window has some configuration options which can be opened as illustrated above. The elements to be changed are:

**Show standard error (SE) in table** Include a column with the % standard error for each parameter.

vB	SE_vB	K1	SE_K1	K2	SE_K2	K3	SE_K3
0.05	NaN	0.067964	4.9342	0.125916	19.4044	0.026781	34.9827
0.05	NaN	0.067964	4.9342	0.125916	19.4044	0.026781	34.9827
0.05	NaN	0.069932	1.5686	0.13934	6.3284	0.037444	25.9652
0.045529	2.441	0.072562	1.5623	0.15751	6.0283	0.052887	19.5018
0.043419	2.3594	0.073413	1.3839	0.16245	5.3518	0.056237	16.5101
0.043474	3.5922	0.073424	2.5735	0.164586	10.1533	0.06053	29.352
0.043408	2.2296	0.073498	1.3719	0.164605	5.3149	0.05902	16.1573

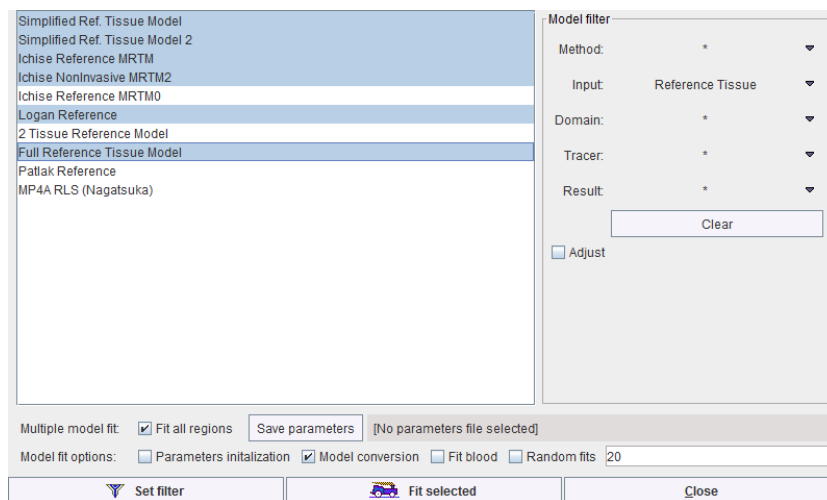
**Show distribution statistics** Enable the lower area showing the **average**, **median** and **stdv**

<b>in table</b>	per column.
<b>Display parameters curve</b>	Shows an additional panel for the visualizing the parameters of the selected rows as curves, as described <i>below</i> (on page 97).
<b>Display relative change curve</b>	As <b>Display parameters curve</b> , but with normalization to the last value.
<b>Region filter</b>	Not applicable here.
<b>Model filter</b>	For filtering the entries listed. With <b>*Comp*</b> , for instance, only compartment model results will be listed.

## Fitting Multiple Models

Using the *model filtering* (on page 62) approach multiple models can sequentially be fitted to a regional TAC, whereby the results are recorded using the *history* (on page 68) mechanism and can easily be inspected.

To specify the fitted models please open the filter dialog window, optionally specify a **Model filter** to reduce the number of models, and select the models of interest as illustrated.



## Initial Parameters

The models will be fitted according to their order in the list. The initial parameters and the fitting flags will correspond to the model default, if the model has not yet been fitted for the regional TAC. Otherwise, the last used parameter configuration will be used (per region) as the starting condition.

Note that configurations on the **Extras** pane are mirrored in the **Model fit options** section. The mainly relevant ones are **Parameters initialization** to estimate initial parameters of compartment models by the linear least squares method, and **Model conversion** which directs the program to apply built-in rules when switching to the next model. This behavior is required for the model set illustrated above, because **SRTM2**, **MRTM2** and **Logan**

**Reference** require the specification of a  $k_2'$  input parameter. With **Model conversion** on, SRTM2, MRTM2 and Logan Reference will inherit the estimated  $k_2'$  from the immediately prior reference tissue model. This can easily be verified by opening the model fitting history.

No	Type	Date & Time	Region	Model	Fitted $k_2$	$k_2'$	BPnd
8	Fit	2013.10.10 20:56:26	putamen	Simplified Ref. Tissue Model	1.064047	0.053322	1.659632
9	Fit	2013.10.10 20:56:26	putamen	Simplified Ref. Tissue Model 2	1.064047	0.053322	1.659632
10	Fit	2013.10.10 20:56:26	putamen	Ichise Reference MRTM		0.053954	1.656537
11	Fit	2013.10.10 20:56:26	putamen	Ichise Noninvasive MRTM2		0.053954	1.656537
12	Fit	2013.10.10 20:56:26	putamen	Logan Reference		0.053954	1.695364
13	Fit	2013.10.10 20:56:26	putamen	Full Reference Tissue Model	1.198914		1.616432

## Fitting all Regions

The multi-model fit can be applied to all regions by enabling the **Fit all regions** box. In this case, as a convenience for the results comparison, the parameters of all model fits in all regions can be directed to a kinetic parameter file (**.kinPar**). Such a file can easily be opened in programs such as MS Excel, but it can also be opened in the **aggregation** (on page 94) window for extracting the information of interest.

## Fitting Problems

In the case of a failed fit it is worthwhile to check the console output. For example, the output text reproduced below clearly indicates that the 4th fit parameter caused a problem.

```
Singular Matrix-2
covar[1][1] = 2.1652603147908747E-6
covar[1][2] = -5.120936302955895E-4
covar[1][3] = 2.862241687078937E-7
covar[1][4] = 0.0
covar[2][1] = -5.120936302955895E-4
covar[2][2] = 0.3918087002588389
covar[2][3] = -2.0406232308544742E-4
covar[2][4] = 0.0
covar[3][1] = 2.862241687078937E-7
covar[3][2] = -2.0406232308544742E-4
covar[3][3] = 1.4044964781403112E-7
covar[3][4] = 0.0
covar[4][1] = 0.0
covar[4][2] = 0.0
covar[4][3] = 0.0
covar[4][4] = 0.0
```

Such problems might be caused by initial values very close to parameter restrictions.



## Coupled Fitting for Improving Parameter Estimates

In some situations the introduction of prior knowledge can help improving the stability of model fitting. In receptor experiments it may be adequate to assume that the distribution volume ( $K_1/k_2$ ) of the non-displaceable compartment and/or the dissociation rate from specific binding sites ( $k_4$ ) are equal in certain tissues of the brain. Hence, a simultaneous fit of selected regional TACs of a scan can be performed which finds separate  $K_1$  and  $k_3$  values per TAC, but which delivers a  $K_1/k_2$  and  $k_4$  which is common for all TACs. As an example, Sanabria-Bohorquez et al [1] have used a coupled  $k_4$  for solving the problem of unstable fits. In PKIN this type of fitting is called **Coupled Fitting**.

The concept can be extended to the fitting of data from different scans of the same subject, for instance test/retest scans which are replicate measurements under the same conditions and where the parameters should be very similar if the regions are consistently outlined. Plisson et al. [2] have applied simultaneous fitting with the data of pigs which were studied in a baseline condition and with different degrees of receptor blocking by cold compound. By applying the simultaneous fitting, they were able to get stable estimates, which was not possible with independent fitting. In PKIN this type of fitting is called **Coupled Studies Fitting** and distinguishes two variants of coupling. **GLOBAL** coupling of a parameter results in single value which is common for all fitted TACs. **REGIONAL** coupling of a parameter results in common value per region. For instance, if  $k_3$  is defined as a parameter with **REGIONAL** coupling and the coupled regions from two scans are *caudate*, *putamen* and *frontal*, a different  $k_3$  estimate will result for each of these three regions, but each of them is common across the TACs of the two scans.

The requirements for coupled fitting in PKIN are:

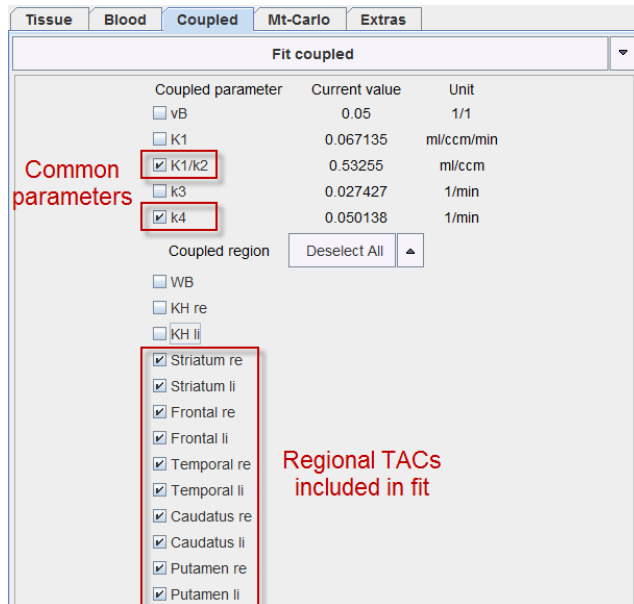
- ▶ The same kinetic compartment model applies to the TACs of several tissue regions (non-compartment models may not be coupled).
- ▶ There exists one or more parameters in the model configuration which can be assumed to have a common value in the model for different TACs.

### References

- [1] Sanabria-Bohorquez SM, Hamill TG, Goffin K, De Lepeleire I, Bormans G, Burns HD, Van Laere K. Kinetic analysis of the cannabinoid-1 receptor PET tracer [(18)F]MK-9470 in human brain. Eur J Nucl Med Mol Imaging. 2010;37(5):920-33. DOI <http://dx.doi.org/10.1007/s00259-009-1340-5>
- [2] Plisson C, Gunn RN, Cunningham VJ, Bender D, Salinas CA, Medhurst AD, Roberts JC, Laruelle M, Gee AD. 11C-GSK189254: a selective radioligand for in vivo central nervous system imaging of histamine H3 receptors by PET. J Nucl Med. 2009;50(12):2064-72. DOI <http://dx.doi.org/10.2967/jnumed.109.062919>

## Common Parameters across Regional TACs of a Single Study

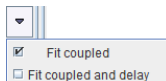
For a simultaneous fit of TACs from a single study the user specifies the coupling on the **Coupled** pane:



In the upper section the kinetic model parameters are listed. Check those parameters which are to be coupled among regions. The lower section lists all selectable regional TACs (having the same model). Check all regions which are to be included in coupled fitting.

**Fit coupled** starts the fitting process as follows:

- » A global target curve is created by stacking the TACs of the checked regions (valid points only).
- » The corresponding weights are calculated according to the weighting configuration for each TAC.
- » A table of fitted parameters is formed by entering the common parameters once and adding all the other fit enabled parameters in the coupled regions. The initial values of common parameters are taken from the first checked region, for the other parameters from the individual regional models. If **Fit blood parameters** is enabled on the **Fitting** pane, the fit-enabled parameters of the blood curves are also taken into account. As a convenience there is a button **Fit coupled and delay**



to estimate a blood delay which considers the information from all coupled regions.

- » The optimizer calculates the model curve in all checked regions, creates the global result curve corresponding to the target curve, weighs all the residuals and forms the global sum of squares as the figure of merit. The parameters are adjusted until the cost function has been minimized, and the resulting parameters are copied back to the individual models.

**Note:** Coupled fitting may take significant processing time. Thus the **Batch mode** facility may be a helpful option to run such jobs at less busy times.

## Common Parameters across Regional TACs from Different Studies

### Preparations

For a simultaneous fit of TACs from multiple studies the user first has to

- » load all of the data sets so that they are available on different tabs;
- » ensure that all of the TACs to be coupled have the same compartment model configuration and the same residual weighting;
- » ensure that all model parameters have reasonable initial values by either fitting the models, or propagating a model configuration.

The coupling is performed on a separate dialog window which can be opened from the **Kinetic** menu with the **Coupled Studies Dialog** entry.

### Study and Model Selection

The dialog window opens on the first tab **Study/Model selection** containing three sections.

Study/Model selection			
Study selection			
Sel	Study	Patient	Description
<input checked="" type="checkbox"/>	Study 1	PKIN1	PKIN1 [CPFPX Bolus] CPFPX Bolus Dynamic PET
<input checked="" type="checkbox"/>	Study 2	PKIN1	PKIN1 [CPFPX Bolus] CPFPX Bolus Dynamic PET
<input checked="" type="checkbox"/>	Study 3	PKIN1	PKIN1 [CPFPX Bolus] CPFPX Bolus Dynamic PET

Model selection	
Model	
1	Tissue Compartment
2	Tissue, K1/k2

Parameters selection		
No	Parameter	Coupling
1	vB	NOT COUPLED
2	K1	NOT COUPLED
3	K1/k2	REGIONAL
4	k3	NOT COUPLED
5	k4	NOT COUPLED
		REGIONAL
		NOT COUPLED
		GLOBAL

0 %

Fit coupled Copy to Clipboard Use results Close

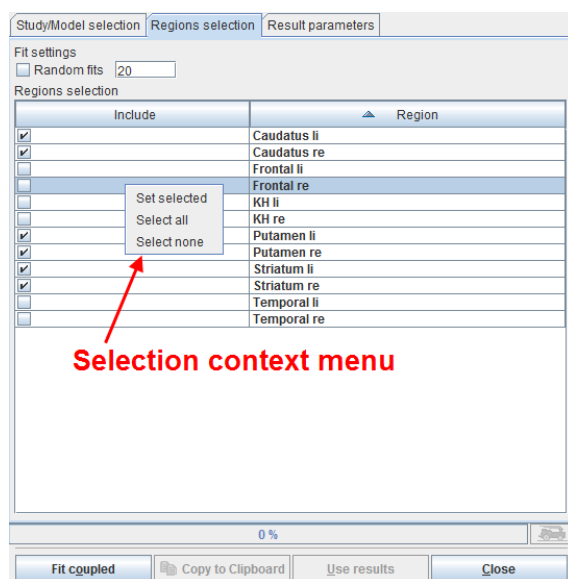
The **Study selection** lists all open data sets. As soon as one of them is selected in the list, the **Model selection** list is updated, showing the different models which are used for the TACs of the study. In the example shown above one of the regions (which is not used for coupling) uses a **1 Tissue Compartment** model, whereas the regions intended for coupling use the **2 Tissue K1/k2** model.

After selecting the appropriate model, the **Parameters selection** list is updated showing all parameters of the model together with a **Coupling** selection. A parameter can have one out of three configurations:

- » **NOT COUPLED:** The parameter will be individually fitted in the different regional models or kept fix depending on the state of the individual fit box.
- » **REGIONAL:** The parameter will be coupled among all regions with the same name. For instance, the parameter will have a common value for all *caudate* regions, a different common one for all *putamen* regions, etc. Note that with this configuration the state of the individual parameter fit boxes is not relevant, the parameter will be modified in all coupled regions.
- » **GLOBAL:** The parameter will be coupled among all coupled regions independent of their name. For instance, the parameter will have the same common value among all *caudate*, *putamen*, etc regions. Note that with this configuration the states of the individual parameter fit boxes is not relevant, the parameter will be modified in all coupled regions.

## Regions Selection and Fitting

The next step is defining the TACs included in the coupled fit on the **Regions selection** pane as illustrated below.



The list contains all regions which are defined in the coupled studies with *exactly the same names*. Initially all regions are selected for inclusion. Regions can selectively be unselected by checking their box. When working with large number of regions (e.g. from atlas definitions) the context menu may be helpful which can be opened by right clicking into the list. It contains **Select all** and **Select none** for quickly selecting or de-selecting all regions, respectively.

Coupled fitting is started using the **Fit coupled** button and proceeds as follows:

- » A global target curve is created by stacking the TACs of all coupled regions from the selected studies (valid points only).
- » The corresponding weights are calculated according to the weighting configuration for each TAC.
- » A table of fitted parameters is formed by entering the **GLOBAL** parameters once, the **REGIONAL** parameters once per region, and adding all the other fit enabled parameters in the coupled regions. The initial values of common parameters are taken from the first

study, for the other parameters from the individual regional models. Note that **Fit blood parameters** has no impact on fitting, all the parameters of the blood curves remain fixed.

The optimizer calculates all model curves, creates the global result curve corresponding to the target curve, weighs all the residuals and forms the global sum of squares as the figure of merit. The parameters are adjusted until the cost function has been minimized. If the box **Random Fits** is checked, the fitting will be repeated as many times as specified with randomly changed starting parameters and the result with minimal global chi square returned.

## Result Parameters

After the coupled fit has completed the resulting parameters in the coupled regions are listed on the **Result parameters** pane which is immediately shown.

REGIONAL GLOBAL

Study/Model selection		Regions selection		Result parameters							
No	Study	Region	Model	vB	K1	K1/k2	k3	k4	Vs	Vt	k3
1	2	Striatum li	2 Tissue, K1/k2	0.05	0.0758...	0.871893	0.0057...	0.0272...	0.1839	1.0557...	0.2109
2	2	Striatum re	2 Tissue, K1/k2	0.05	0.0713...	0.841836	0.0044...	0.0272...	0.1363	0.9781...	0.1619
3	2	Caudatus li	2 Tissue, K1/k2	0.05	0.07054	0.803028	0.0071...	0.0272...	0.2096...	1.0126...	0.2610
4	2	Putamen re	2 Tissue, K1/k2	0.05	0.0776...	0.913206	0.0042...	0.0272...	0.1432...	1.0564...	0.1566
5	2	Putamen li	2 Tissue, K1/k2	0.05	0.0788...	0.928673	0.0045...	0.0272...	0.1550...	1.0837...	0.1670
6	2	Caudatus re	2 Tissue, K1/k2	0.05	0.0594...	0.748393	0.00373	0.0272...	0.1024...	0.8508...	0.1366
7	1	Striatum li	2 Tissue, K1/k2	0.05	0.0754...	0.871893	0.0056...	0.0272...	0.1822...	1.05418	0.2099
8	1	Striatum re	2 Tissue, K1/k2	0.05	0.0713...	0.841836	0.0045...	0.0272...	0.1381...	0.9810...	0.1653
9	1	Caudatus li	2 Tissue, K1/k2	0.05	0.0703...	0.803028	0.0070...	0.0272...	0.2080...	1.0110...	0.2599
10	1	Putamen re	2 Tissue, K1/k2	0.05	0.0775...	0.913206	0.0043...	0.0272...	0.1472...	1.0604...	0.1612
11	1	Putamen li	2 Tissue, K1/k2	0.05	0.0783...	0.928673	0.0044...	0.0272...	0.1527...	1.0814...	0.1644
12	1	Caudatus re	2 Tissue, K1/k2	0.05	0.0595	0.748393	0.0038...	0.0272...	0.1049...	0.8533...	0.1402
13	3	Striatum li	2 Tissue, K1/k2	0.05	0.0755...	0.871893	0.0056...	0.0272...	0.1818...	1.0537...	0.2085
14	3	Striatum re	2 Tissue, K1/k2	0.05	0.07144	0.841836	0.0043...	0.0272...	0.1346...	0.97646	0.1599
15	3	Caudatus li	2 Tissue, K1/k2	0.05	0.0703...	0.803028	0.0068...	0.0272...	0.2025...	1.00555	0.2521
16	3	Putamen re	2 Tissue, K1/k2	0.05	0.07766	0.913206	0.0042...	0.0272...	0.1407...	1.0539...	0.1541
17	3	Putamen li	2 Tissue, K1/k2	0.05	0.0782...	0.928673	0.0045...	0.0272...	0.1545...	1.0832...	0.1664
18	3	Caudatus re	2 Tissue, K1/k2	0.05	0.0596...	0.748393	0.0037...	0.0272...	0.1018...	0.85022	0.1366

No	Study	Statistics	Model	vB	K1	K1/k2	k3	k4	Vs	Vt
1	Statistics	average		0.05	0.0721...	0.851172	0.0049...	0.0272...	0.1545...	1.0056...
2	Statistics	median		0.05	0.0734...	0.856865	0.0044...	0.0272...	0.15001	1.0331...
3	Statistics	stdv		1.3877...	0.0063...	0.062253	0.0010...	0.0	0.0332...	0.0769...

Fit coupled

Copy to Clipboard

Use results

Close

Note in the example above that the **REGIONAL** parameter  $K_1/k_2$  has the same value in all regions with the same name, whereas the **GLOBAL** parameter  $k_4$  has the same value in all regions.

The results can be prepared for use in another program such as MS Excel with the **Copy to Clipboard** button. The **Use results** button transfers the result parameters together with their standard errors to the individual models and closes the dialog window, whereas the results are discarded by the **Close** button.

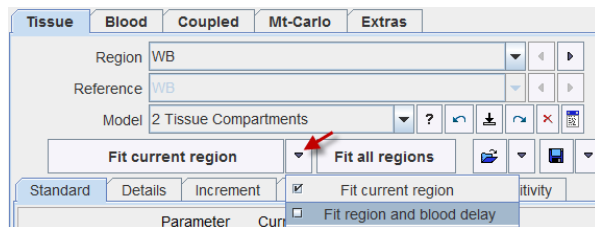
## Blood Delay Fitting

The timing of the tissue and the blood data should be synchronized to a common clock. Nevertheless, there may be an inherent shift of the blood activity curves relative to the tissue activity curves because of the arrival time difference of the blood in the target tissue and at the sampling location. For brain PET scans with radial artery blood sampling the tracer will appear later in the blood samples than in the brain. This delay is even prolonged if the blood samples are not taken directly with a syringe, but via a catheter which adds additional pathlength as required by online blood sampling devices.

Such a relative delay of the blood samples is accounted for by the **Delay** parameter of the **blood models** (on page 52). Positive delays represent delayed blood information and hence shift the blood curves to earlier times (to the left). While both the whole-blood and the plasma model include a separate **Delay** parameter, it is reasonable to assume a common delay since the plasma activity is derived from whole-blood. Such a common delay can be fitted together with the kinetic parameters of the tissue model as described below.

### Delay Fitting with Single Tissue TAC

The **Tissue** tab features a dedicated **Fit region and blood delay** button. It is hidden underneath the **Fit current region** button and can be activated as illustrated below.



When it is activated, the following actions are performed:

- » The state of the fit flags of the **Whole blood** and **Plasma** models is saved.
- » The fit flag of the **Delay** is enabled, and all other fit flags disabled.
- » A fit of the tissue model is performed, whereby the **Delay** (common to plasma and whole blood) is also optimized.
- » The state of the fit flags of the **Whole blood** and **Plasma** models is restored.
- » The estimated **Delay** can be inspected on the **Blood** tab or in the model history.

The illustration below shows a history of the outcome when fitting a 1- and 2-Tissue compartment model to the WB TAC including the delay.

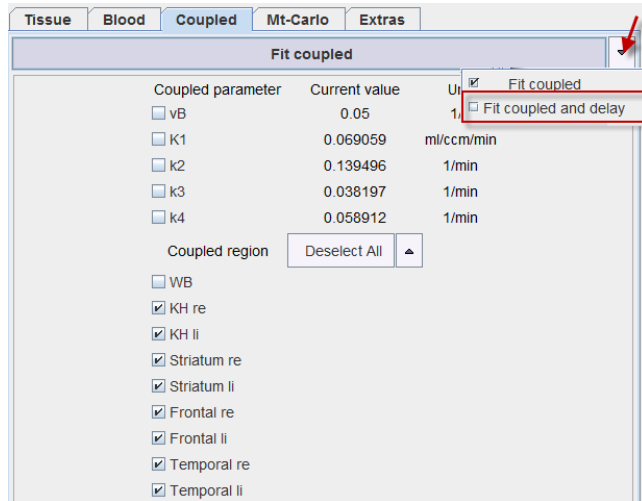
No	Type	Region...	Model	Blood delay	vB	K1	k2	k3	k4	Vt
1	Fit	WB	1 Tissue Compartment	0.0	0.05	0.061412	0.063889			0.7320
2	Fit	WB	1 Tissue Compartment	5.138192	0.05	0.062031	0.085247			0.7276
3	Fit	WB	2 Tissue Compartments	5.412845	0.05	0.069059	0.139496	0.038197	0.058912	0.8160

It is notable that the optimal delay depends on the fitted model, and it will also depend on the selected regional TAC. When using a single TAC it is recommended using the signal from a relatively large VOI, for instance the whole brain, and then keep the delay fixed.

**Note:** A change of the blood **Delay** affects the model curve of all regions and therefore requires re-fitting all regional models.

## Delay Fitting with Multiple Tissue TACs

An alternative to using a single representative tissue is the use of *coupled fitting* (on page 73) for the determination of a blood delay which is optimal with respect to tracer appearance in several regions. In the example below 8 regions are selected on the **Coupled** pane all of which have a 2-tissue compartment model configured.



Fitting of the common delay is started using the **Fit coupled and delay** button which is hidden under the **Fit coupled** button. The procedure will fit all 8 compartment models while at the same time optimizing the blood timing.

## Assessing Fit Results

The physiologic behavior of PET tracers is usually quite complex. A comprehensive model accurately describing its kinetics therefore requires many compartments and exchange parameters. However, as the PET signal is limited in quality and information, the model must be simplified to a degree that only a few parameters remain. When estimating a model with many parameters, the variance of the parameter estimates tends to be very high, so that a reliable interpretation of the results becomes impossible. A simplified model with fewer parameters will provide more precise results, but these parameters may be biased. Therefore, the optimal trade-off between bias and uncertainty has to be sought by testing models of decreasing complexity.

After every model fit detailed information about the parameter estimates and the goodness-of-fit is available on the **Details** tab.

Standard	Details	Increment	Restrictions	Weighting	Sensitivity
Parameter	Current value	% SE	Conf.low	Conf.high	Unit
vB	0.05	---	---	---	1/1
K1	0.067135	5.0	0.06028	0.07399	ml/ccm/min
k2	0.126062	19.71	0.07532	0.1768	1/min
k3	0.027427	94.56	-0.02553	0.08039	1/min
k4	0.050138	67.91	-0.01939	0.11967	1/min
Vs	0.291315	29.23	0.11746	0.46517	ml/ccm
Vt	0.823866	7.27	0.70152	0.94621	ml/ccm
K1/k2	0.53255	15.87	0.35998	0.70512	ml/ccm
k3/k4	0.547019	42.45	0.07282	1.02122	1/1
Flux	0.011996	66.51	-0.0043	0.02829	ml/ccm/min
Parameter	Current value				
DOF	30.0				
SumSquared	4.485386				
ChiSquare	7.010935				
AIC	70.958472				
SC	76.063914				
MSC	2.788165				
R2	0.951367				
Sy.x	0.386669				
Runs test	1.0				
AUC	11122.199461				

### Parameter Confidence Intervals

Nonlinear regression using the Marquart-Levenberg optimization reports a standard error for each fitted parameter value. If the inherent fitting assumptions are true, a 95% confidence interval can be approximated by the result parameter plus/minus two standard errors. This confidence interval is displayed for each fitted parameter (**Conf.low**, **Conf.high**).

There will be a 95% chance that the confidence interval contains the true parameter value. A sufficiently narrow confidence interval indicates that the parameter could be determined with a reasonable certainty, whereas a wide interval makes it necessary to revise the configuration of the used model, or look for a more appropriate model.

### Goodness-of-Fit

The following measures are defined which allow a direct or indirect assessment of the goodness-of-fit:

**DOF** **Degrees of freedom** defined as the number of valid measurements minus the number of fitted parameters.

**SumSquared** Sum of squared (unweighted) residuals.



<b>ChiSquare</b>	<p><b>Reduced Chi square.</b> Sum of squared, weighted residuals, divided by the degrees of freedom.</p> <p>The reduced chi-square provides a useful measure of goodness-of-fit. If the model describes the measured data, the reduced chi-square will mostly represent the variance of the data and will be close to 1.0 (when weighting is appropriate).</p>
<b>AIC</b>	<p><b>Akaike Information Criterion.</b> The AIC methodology attempts to find the model that best explains the data with a minimum of free parameters. The AIC is calculated with the second order correction for small sample size (&lt;40).</p> <p>The preferred model is the one with the lowest AIC value.</p>
<b>SC</b>	<p><b>Schwartz Criterion</b>, also called Bayesian Information Criterion (BIC).</p> <p>The preferred model is the one with the lowest SC value.</p>
<b>MSC</b>	<p>Another criterion used in the <i>Scientist</i> Software (MicroMath, Saint Louis, Missouri USA) is the <b>Model Selection Criterion</b>.</p> <p>The preferred model is the one with the highest MSC value.</p>
<b>R2</b>	<p>There is also a measure of the goodness-of-fit, the <b>coefficient of determination R2</b>, a number between 0 and 1. A value of 0 means that the fit is not better than a horizontal line through the mean of all measurements, whereas a value of 1 means that all measurements lie exactly on the curve. High R2 values indicate that the model curve is close to the measurement.</p>
<b>Sy,x</b>	<p>Another information about the residuals is provided by the <b>root mean square value Sy,x</b>. It is defined as the standard deviation of the residuals and can be used to generate synthetic measurements in Monte Carlo simulations, provided all measurements have the same variability.</p>
<b>Runs test</b>	<p>The runs test is a statistical test to decide whether the model curve deviates systematically from the data. It is based on the number of runs resulting from the fit. A run is a set of consecutive measurements which are above (positive residuals) or below (negative residuals) the measurement. Given the assumption that the residuals are randomly distributed, the probability <math>p</math> of the occurrence of a number of runs can be calculated. If <math>p</math> is small (eg. <math>p &lt; 0.05</math>) the measurements systematically deviate from the model curve. Such a finding signals that most likely an inadequate model was fitted and further investigations of the result are not sensible.</p> <p><b>1</b> indicates a systematic deviation between model and measurement (<math>p &lt; 0.05</math>). <b>0</b> indicates that there is no significant deviation.</p>
<b>AUC</b>	<p>Area under the model curve.</p>

## Criteria for Comparing Models

The following criteria can be used for comparing different models which use the same weighting and are fitted to the same data. The notation is:

- n: Number of independent measurements considered in the fit
- p: Number of fitted parameters
- w<sub>i</sub>: Weight applied to residual of acquisition i

### Akaike Information Criterion (AIC)

The Akaike Information Criterion [1] is defined by the formula

$$AIC = n \ln \left( \sum_i w_i [y(t_i) - \hat{y}(t_i)]^2 / n \right) + 2p$$

If applicable, PKIN uses an adjusted procedure with a second order correction for small sample size (<40) [2]

$$AIC = n \ln \left( \sum_i w_i [y(t_i) - \hat{y}(t_i)]^2 / n \right) + 2p + \frac{2p(p+1)}{n-p-1}$$

The more appropriate model is the one with the *smaller* AIC value.

### Schwartz Criterion (SC)

The Schwartz Criterion is defined by the formula

$$SC = n \ln \left( \sum_i w_i [y(t_i) - \hat{y}(t_i)]^2 / n \right) + p \ln(n)$$

The more appropriate model is the one with the *smaller* SC value.

### Model Selection Criterion

Another criterion used in the *Scientist* Software (MicroMath, Saint Louis, Missouri USA) is the **Model Selection Criterion**

$$MSC = \ln \left( \frac{\sum_i w_i [y(t_i) - \bar{y}(t_i)]^2}{\sum_i w_i [y(t_i) - \hat{y}(t_i)]^2} \right) - 2p/n$$

This criterion has the advantage that it is independent of the magnitude of the y<sub>i</sub>. Opposed to the AIC and the SC the more appropriate model is that with the *larger* MSC.

### F-Test

Two nested models can be compared by an F-test of their sum of squared residuals [3]. The idea is to compare the total sum of squares into a component removed by the simpler model

and into a component additionally removed by the more complex model. For each component, the mean square (sum of squares per degree of freedom) is calculated. The residual mean square is an estimate of the variance of the original data. The ratio of the two mean squares is the F-statistic used to test for significance of the variance reduction by the additional parameters, as follows:

$$F = \frac{(Q_1 - Q_2)/(p_2 - p_1)}{Q_2/(n - p_2)}$$

where

$Q_1$  represent the sum of squares or the simple model with  $p_1$  parameters,

$Q_2$  the sum of squares or the more complex model with  $p_2$  parameters,

$p_1 < p_2$

The F-statistics has  $(p_2 - p_1, n - p_2)$  degrees of freedom. If the calculated F is larger than the tabulated value at a specified p value, the reduction of the residual variation by the addition of the  $(p_2 - p_1)$  extra parameters of the more complex model is statistically significant.

Usually, significance  $p=0.05$ .

Note: in MS Excel the  $F_{p_2-p_1, n-p_2}$  can be calculated by the function FINV( $p; p_2-p_1, n-p_2$ ).

## References

- [1] Akaike, H. A new look at the statistical model identification. IEEE Trans. Automat. Contr 1974, AC19: 716-723.
- [2] Motulsky H, Christopoulos A. Fitting Models to Biological Data using Linear and Nonlinear Regression. New York: Oxford University Press; 2004.
- [3] Carson RE, Parameter Estimation in PET, In: M. Phelps JM, and Schelbert H, eds. Positron Emission Tomography and Autoradiography: Principles and Applications for the Brain and the Heart., New York: Raven Press; 1986: 287-346.

## Analysis of Sensitivity and Identifiability

The following two approaches can be employed to investigate whether a selected model is too complex for the data. The implementation in PKIN is based on the handout *Kinetic model evaluation with sensitivity functions and correlation matrices* by Dr. M.M. Graham, University of Washington at the Technical Exhibition of the Society of Nuclear Medicine Annual Meeting 1995 (JNM vol 36, no 5, P1208). As to our knowledge, there is no follow-up paper about this subject.

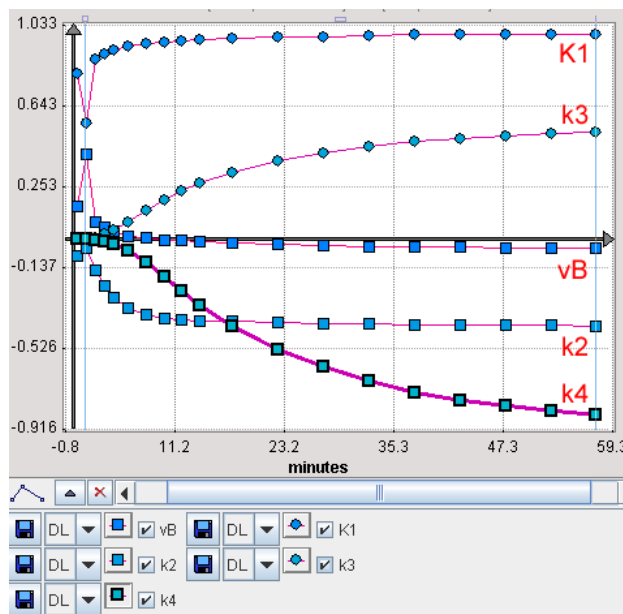
### Sensitivity

If a parameter value is changed there should be some visual change in the model output. Some parameters will cause more change than other. The model is more sensitive to a parameter that causes more change in the output.

The sensitivity functions  $Sens(k_i)$  for the different parameters  $k_i$  are obtained by calculating the model function twice: with the initial parameter set, and after changing the parameter  $k_i$  by 1%. Then, the expression

$$Sens(k_i, t) = 100 * [Model(k_i * (1.01), t) - Model(k_i, t)] / Model(k_i, t)$$

is calculated over time. When the 1% change of  $k_i$  has changed the output by 1% at a certain time the sensitivity function is equal to 1. In PKIN, the sensitivity functions are calculated for all model parameters, whenever the **Sensitivity** pane is selected. The example below shows the sensitivity functions of a FDG data set.



Examination of the sensitivity curves shows at what time a parameter has significant sensitivity and how sensitive a parameter is compared to another. If the sensitivity curves of two parameters have the same shape it is unlikely that one can tell which parameter has changed (ie. they are not identifiable).

In the FDG example above the model is quite insensitive to  $vB$ . Furthermore, the  $K_1$  and  $k_2$  curves have a quite similar (but opposite) shape throughout the acquisition duration, indicating that there is a substantial correlation between these parameters.

## Identifiability

The model output must change in a unique way when changing a model parameter. If the change in the model output is the same when changing either of two parameters they can not be independently identified. In this situation the model must be simplified further, or one of the parameters can be fixed at a physiologic value. The latter approach is better since simplification usually means essentially setting one of the parameters equal to zero or infinity.

A more quantitative way to look at parameter identifiability is to generate a correlation matrix. It shows how the various parameters tend to correlate with each other. A low value, close to 0, means they do not correlate and that they are identifiable from one another. A high value, close to 1, means they are highly correlated and not independently identifiable.

The correlation matrix is calculated by:

- 1) generating the sensitivity matrix SM given from the sensitivity functions by  

$$SM(i,j) = \text{Integral}(\text{Sens}(k_i, t) * \text{Sens}(k_j, t));$$

inverting the sensitivity matrix  $SM(i,j)$ , resulting in the covariance matrix;

normalizing the covariance matrix by dividing each element by the square root of the product of the corresponding diagonal elements; result is the correlation matrix .

The correlation matrix is shown in the **Sensitivity** pane. Using the **Save** button the correlation matrix and the sensitivity functions can be save into a text file.

Details	Increment	Restrictions	Sensitivity						
%Param. change		<input type="text" value="1.0"/>							
Correlation matrix		vB	K1	k2	k3	k4			
vB		1.0	-0.2457	-0.1473	-0.0104	0.3431			
K1		-0.2457	1.0	0.8861	0.6102	0.2438			
k2		-0.1473	0.8861	1.0	0.8864	0.5444			
k3		-0.0104	0.6102	0.8864	1.0	0.8359			
k4		0.3431	0.2438	0.5444	0.8359	1.0			
Save									

**Note:** Sensitivity functions are currently only supported for compartment models.

## Comparison in an Additional Window

PKIN allows to examine different models and compare their numerical output using the **History** (on page 68) mechanism. Sometimes it is also helpful to compare the model curves resulting from different model configurations in question. This can be achieved using the **Study Compare Dialog** in the **Kinetic**.

A new dialog window is opened with a clone of the current data set. It supports all features of PKIN except for the functions related to data management. So the user can arrange the original PKIN window and the dialog window next to each other and process/visualize the same data in parallel.

## Parameter Overview

A convenient overview of the current models and their parameters can be obtained using **View Par** in the **Tools** selection (lower right). It shows the same dialog window as the **Fitting History** (on page 68), except that the list entries correspond to the different TACs of the data set. The column maximum is highlighted in red, the minimum in green. Values can be sorted by clicking into the column headers, and the parameter order changed by dragging the column headers left/right. Data export is facilitated via **Copy to Clipboard**.

Parameters																
Display parameters																
Display relative change curve																
No	Region	Model	Blood delay	vB	K1	k2	k3	k4	Vs	Vt	K1/k2	k3/k4	Flux	DOF	SumSquared	ChiSquare
1	Whole Brain	2 Tissue Comp...	0.0	0.05	0.067964	0.125916	0.026781	0.050104	0.288503	0.8282...	0.539756	0.534506	0.01192	30.0	4.501088	6.921222
2	Cerebellum re	2 Tissue Comp...	0.0	0.05	0.080675	0.227288	0.018878	0.040105	0.167083	0.5220...	0.354947	0.470726	0.006187	30.0	4.376307	8.920372
3	Cerebellum li	2 Tissue Comp...	0.0	0.05	0.079204	0.225141	0.018976	0.042646	0.156536	0.5083...	0.351796	0.444963	0.006157	30.0	5.199246	10.926601
4	Striatum re	2 Tissue Comp...	0.0	0.05	0.075372	0.08494	0.003673	0.026039	0.12517	1.0125...	0.887359	0.14106	0.003124	30.0	8.564676	9.732701
5	Striatum li	2 Tissue Comp...	0.0	0.05	0.082455	0.09473	0.009473	0.040261	0.204793	1.0752...	0.870414	0.235282	0.007496	30.0	7.427163	7.41778
6	Caudatus re	2 Tissue Comp...	0.0	0.05	0.06773	0.081497	0.003049	0.023789	0.106521	0.9375...	0.831067	0.128174	0.002443	30.0	10.33004	13.800211
7	Caudatus li	2 Tissue Comp...	0.0	0.05	0.075476	0.090835	0.008503	0.03348	0.211025	1.04194	0.830915	0.253968	0.00646	30.0	7.63346	8.528258
8	Putamen re	2 Tissue Comp...	0.0	0.05	0.085583	0.08794	0.003563	0.02276	0.152368	1.1255...	0.973205	0.156563	0.003333	30.0	9.123137	8.521624
9	Putamen li	2 Tissue Comp...	0.0	0.05	0.087752	0.092447	0.0068	0.039029	0.165379	1.1145...	0.949216	0.174227	0.006012	30.0	8.834322	8.021594
10	Frontal re	2 Tissue Comp...	0.0	0.05	0.077295	0.107137	0.015549	0.046244	0.242585	0.9640...	0.721462	0.336242	0.009796	30.0	4.739084	5.560145
11	Frontal li	2 Tissue Comp...	0.0	0.05	0.078394	0.113647	0.020608	0.05436	0.261509	0.9513...	0.689806	0.379105	0.012034	30.0	4.111366	4.814794
12	Temporal re	2 Tissue Comp...	0.0	0.05	0.077227	0.109839	0.027985	0.065548	0.300183	1.0032...	0.703094	0.426946	0.015681	30.0	3.6776	4.150216
13	Temporal li	2 Tissue Comp...	0.0	0.05	0.078609	0.109144	0.024771	0.061313	0.29098	1.0112...	0.720234	0.404007	0.014541	30.0	3.870461	4.273041

No	Region	Model	Blood delay	vB	K1	k2	k3	k4	Vs	Vt	K1/k2	k3/k4	Flu
1		average		0.05	0.0779...	0.1192...	0.0145...	0.0419...	0.2055...	0.9304...	0.7248670...	0.314289...	0.0
2		median		0.05	0.0783...	0.1071...	0.0155...	0.0402...	0.2047...	1.0032...	0.7214615...	0.336241...	0.0
3		stdv		0.0	0.0055...	0.0472...	0.0087...	0.0130...	0.0634...	0.1923...	0.1949638...	0.134304...	0.0

Copy to Clipboard			Select	Close
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Settings

☐ Show standard error (SE) in table

☒ Show distribution statistics in table

☒ Display parameters curve

☒ Display relative change curve (relative to last point)

☐ Region filter

☐ Model filter

Ok Cancel

Note the configuration options which allow showing the standard errors in the table, or visualizing the parameters of the selected rows as curves.

# Special Processing Modes

## Parameter Sensitivity to Acquisition Duration

A standard task when analyzing kinetic data of a new tracer is assess the dependence of the parameters on the length of the available data. This type of analysis can easily be performed using the **Data shortening** option on the **Extras (on page 63)** panel. As soon as this option is enabled with a minimal data duration, the following happens when activating **Fit Current region**:


- 1) The set of initial parameters is created.

A fit is performed with the minimal data segment (e.g. the first 20 min), corresponding to a small number of Degrees Of Freedom (e.g. DOF=16) . All other fit options on the **Extras** panel are also active are active.

An additional sample is added to the data segment and the fit repeated with the same initial parameters (DOF=17).

This step is repeated until the full data segment has been fitted.

## Results

The results are stored in the *model fitting history* (on page 68) and can be opened in the with the PKIN Parameters Explorer with the  button. All fits of the sensitivity fits have the same **Revision** number. The data segment length is represented by the DOF column.

Parameters	Display parameters	Display relative change curve																
No	Revision	Type	Date & Time	Region	Model	Blood delay	v0	K1	k2	k3	k4	Vt	K1/K2	K3/K4	Flux	DOF	Sum Squared	
1	0	Load	2014.07.04 12:24:50	Whole Brain	2 Tissue Comp.	0.0	0.05	0.067954	0.125916	0.026781	0.050104	0.288503	0.8282	0.539756	0.534506	0.01192	30.0	4.501088
2	0	Fit	2014.07.04 12:24:51	Whole Brain	2 Tissue Comp.	0.0	0.02674	0.083802	0.255948	0.136473	0.087879	0.50847	0.8358	0.327419	1.552961	0.029144	29.0	1.225176
3	0	Fit	2014.07.04 12:24:52	Whole Brain	2 Tissue Comp.	0.0	0.05	0.067954	0.125916	0.026781	0.050104	0.288503	0.8282	0.539756	0.534506	0.01192	30.0	4.501088
4	1	Load	2014.10.17 09:41:42	Whole Brain	2 Tissue Comp.	0.0	0.05	0.067954	0.125916	0.026781	0.050104	0.288503	0.8282	0.539756	0.534506	0.01192	30.0	4.501088
5	2	Fit All	2014.10.17 09:45:34	Whole Brain	2 Tissue Comp.	0.0	0.05	0.067954	0.125916	0.026781	0.050104	0.288503	0.8282	0.539756	0.534506	0.01192	30.0	4.501088
6	3	Fit	2014.10.17 11:05:36	Whole Brain	2 Tissue Comp.	0.0	0.05	0.068272	0.128178	0.02323	1.2399	9.97838	9.9783	0.526333	1.87340	0.010475	16.0	4.466595
7	3	Fit	2014.10.17 11:05:37	Whole Brain	2 Tissue Comp.	0.0	0.05	0.068237	0.128616	0.027827	0.033231	0.441089	0.97164	0.530551	0.83138	0.012066	17.0	4.473859
8	3	Fit	2014.10.17 11:05:38	Whole Brain	2 Tissue Comp.	0.0	0.05	0.068352	0.131183	0.035509	0.063317	0.292212	0.8132	0.521044	0.560819	0.014561	18.0	4.478201
9	3	Fit	2014.10.17 11:05:39	Whole Brain	2 Tissue Comp.	0.0	0.05	0.068452	0.132799	0.039496	0.073518	0.276919	0.7923	0.515458	0.537229	0.015692	19.0	4.479817
10	3	Fit	2014.10.17 11:05:39	Whole Brain	2 Tissue Comp.	0.0	0.05	0.068435	0.13266	0.039533	0.07444	0.275984	0.7923	0.515869	0.536956	0.015716	20.0	4.480069
11	3	Fit	2014.10.17 11:05:40	Whole Brain	2 Tissue Comp.	0.0	0.05	0.068441	0.132698	0.039438	0.073764	0.275755	0.7915	0.515766	0.534653	0.015681	21.0	4.480251
12	3	Fit	2014.10.17 11:05:41	Whole Brain	2 Tissue Comp.	0.0	0.05	0.0684	0.13199	0.037793	0.07041	0.27816	0.7963	0.518223	0.536757	0.015226	22.0	4.481149
13	3	Fit	2014.10.17 11:05:42	Whole Brain	2 Tissue Comp.	0.0	0.05	0.068293	0.130481	0.034902	0.065131	0.280473	0.8038	0.523392	0.538876	0.014412	23.0	4.482008
14	3	Fit	2014.10.17 11:05:42	Whole Brain	2 Tissue Comp.	0.0	0.05	0.068317	0.13062	0.0348	0.064818	0.281668	0.80469	0.523022	0.53854	0.014372	24.0	4.483211
15	3	Fit	2014.10.17 11:05:43	Whole Brain	2 Tissue Comp.	0.0	0.05	0.068283	0.130688	0.033703	0.062239	0.284236	0.8091	0.524899	0.541606	0.01405	25.0	4.485458
16	3	Fit	2014.10.17 11:05:43	Whole Brain	2 Tissue Comp.	0.0	0.05	0.068283	0.130159	0.033762	0.061777	0.286624	0.8112	0.524614	0.543652	0.014061	26.0	4.489102
17	3	Fit	2014.10.17 11:05:44	Whole Brain	2 Tissue Comp.	0.0	0.05	0.068147	0.12822	0.030385	0.065329	0.286696	0.8181	0.531485	0.539426	0.013065	27.0	4.492525
18	3	Fit	2014.10.17 11:05:44	Whole Brain	2 Tissue Comp.	0.0	0.05	0.068068	0.127221	0.028789	0.05359	0.287426	0.8224	0.535039	0.537205	0.012561	28.0	4.496269
19	3	Fit	2014.10.17 11:05:45	Whole Brain	2 Tissue Comp.	0.0	0.05	0.068015	0.126542	0.027709	0.051678	0.288192	0.8256	0.537489	0.536181	0.012218	29.0	4.4991
20	3	Fit	2014.10.17 11:05:45	Whole Brain	2 Tissue Comp.	0.0	0.05	0.067954	0.125916	0.026781	0.050104	0.288503	0.8282	0.539756	0.534506	0.01192	30.0	4.501088

Shortening

No	Revision	Type	Date & Time	Region	Model	Blood delay	v0	K1	0.0689	0.1353	0.0368	0.0571	4.9891	0.5197841	9.367038	0.01414	24.7
1	0	average				0.0	0.05	0.0682	0.1293	0.0302	0.0590	0.2678	0.803	0.5277245	0.536981	0.01355	25.6
3	0	std				0.0	0.0050	0.0034	0.0277	0.0233	0.0177	2.147	2.1747	0.044764	0.420996	0.00376	4.76

Copy to Clipboard

Delete

Select

Close

## Sensitivity Visualization

The results can be exported via **Copy to Clipboard**, but the trend can also directly be visualized on the **Display parameters** and **Display relative change curve** panels. The selection in the list defines the data to be visualized:

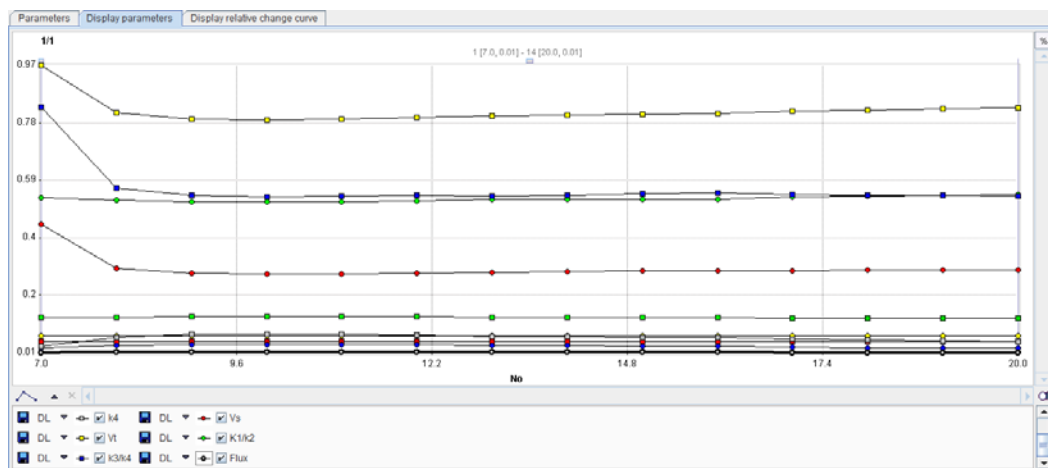
- ▶ Selection of a single line as illustrated above: all entire with the same **Revision** number are used.
- ▶ Explicit selection of a subset of lines as illustrated below. In this particular example the Vt value degenerated for the shortest segment ( $9.98 \times 10^4$ ), which resulted in unusable



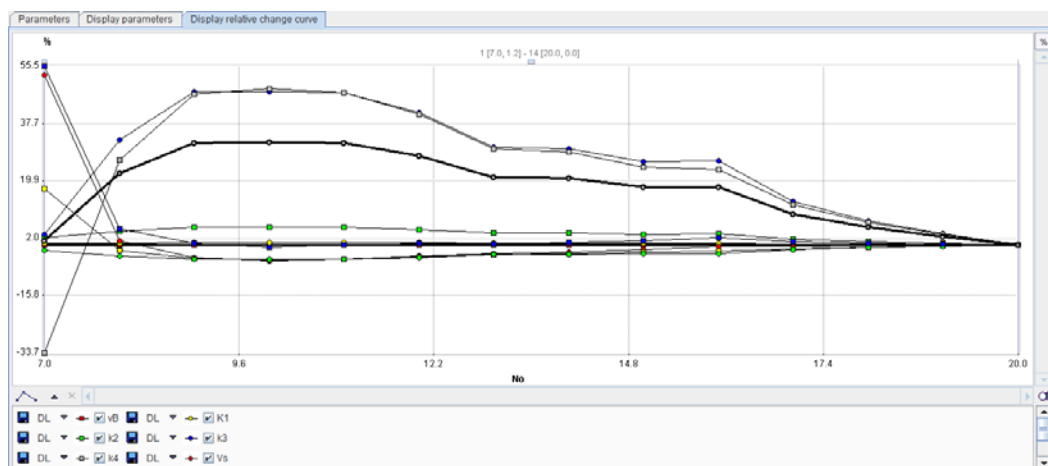
plots. Therefore only the longer segment fits were selected.

No	Revision	Type	Date & Time	Region	Model	Blood delay	vB	K1	K2	K3	K4	Vs	Vt	K1/K2
1	0	Load	2014.07.04 12:24:50	Whole Brain	2 Tissue Comp.	0.0	0.05	0.067964	0.125916	0.026781	0.050104	0.288503	0.826259	0.539756
2	0	Fit	2014.07.04 12:24:51	Whole Brain	2 Tissue Comp.	0.0	0.05	0.067964	0.125916	0.026781	0.050104	0.288503	0.826259	0.539756
3	0	Fit	2014.07.04 12:24:52	Whole Brain	2 Tissue Comp.	0.0	0.05	0.067964	0.125916	0.026781	0.050104	0.288503	0.826259	0.539756
4	1	Load	2014.10.17 09:41:42	Whole Brain	2 Tissue Comp.	0.0	0.05	0.067964	0.125916	0.026781	0.050104	0.288503	0.826259	0.539756
5	2	Fit	2014.10.17 09:45:34	Whole Brain	2 Tissue Comp.	0.0	0.05	0.067964	0.125916	0.026781	0.050104	0.288503	0.826259	0.539756
6	3	Fit	2014.10.17 11:05:36	Whole Brain	2 Tissue Comp.	0.0	0.05	0.067964	0.125916	0.026781	0.050104	0.288503	0.826259	0.539756
7	3	Fit	2014.10.17 11:05:37	Whole Brain	2 Tissue Comp.	0.0	0.05	0.067964	0.125916	0.026781	0.050104	0.288503	0.826259	0.539756
8	3	Fit	2014.10.17 11:05:38	Whole Brain	2 Tissue Comp.	0.0	0.05	0.067964	0.125916	0.026781	0.050104	0.288503	0.826259	0.539756
9	3	Fit	2014.10.17 11:05:39	Whole Brain	2 Tissue Comp.	0.0	0.05	0.067964	0.125916	0.026781	0.050104	0.288503	0.826259	0.539756
10	3	Fit	2014.10.17 11:05:39	Whole Brain	2 Tissue Comp.	0.0	0.05	0.067964	0.125916	0.026781	0.050104	0.288503	0.826259	0.539756
11	3	Fit	2014.10.17 11:05:40	Whole Brain	2 Tissue Comp.	0.0	0.05	0.067964	0.125916	0.026781	0.050104	0.288503	0.826259	0.539756
12	3	Fit	2014.10.17 11:05:41	Whole Brain	2 Tissue Comp.	0.0	0.05	0.067964	0.125916	0.026781	0.050104	0.288503	0.826259	0.539756
13	3	Fit	2014.10.17 11:05:42	Whole Brain	2 Tissue Comp.	0.0	0.05	0.067964	0.125916	0.026781	0.050104	0.288503	0.826259	0.539756
14	3	Fit	2014.10.17 11:05:42	Whole Brain	2 Tissue Comp.	0.0	0.05	0.067964	0.125916	0.026781	0.050104	0.288503	0.826259	0.539756
15	3	Fit	2014.10.17 11:05:43	Whole Brain	2 Tissue Comp.	0.0	0.05	0.067964	0.125916	0.026781	0.050104	0.288503	0.826259	0.539756
16	3	Fit	2014.10.17 11:05:43	Whole Brain	2 Tissue Comp.	0.0	0.05	0.067964	0.125916	0.026781	0.050104	0.288503	0.826259	0.539756
17	3	Fit	2014.10.17 11:05:44	Whole Brain	2 Tissue Comp.	0.0	0.05	0.067964	0.125916	0.026781	0.050104	0.288503	0.826259	0.539756
18	3	Fit	2014.10.17 11:05:44	Whole Brain	2 Tissue Comp.	0.0	0.05	0.067964	0.125916	0.026781	0.050104	0.288503	0.826259	0.539756
19	3	Fit	2014.10.17 11:05:45	Whole Brain	2 Tissue Comp.	0.0	0.05	0.067964	0.125916	0.026781	0.050104	0.288503	0.826259	0.539756
20	3	Fit	2014.10.17 11:05:45	Whole Brain	2 Tissue Comp.	0.0	0.05	0.067964	0.125916	0.026781	0.050104	0.288503	0.826259	0.539756

When selecting the **Display parameters** panel the absolute parameter values are visualized. Each parameter corresponds to a curve and can be enabled/disabled with the checkboxes in the curve control area. The x-axis value corresponds to the number in the **No** column, so the length of the data segment increases to the right.



When selecting the **Display relative change curve** panel the same curves are shown, but the values are normalized to the last point (which in the case of the sensitivity fitting can be considered as the most reliable one).





## Monte Carlo Simulations

The purpose of performing Monte Carlo simulations in kinetic modeling is to get information about the standard error of the parameter values found in a fit. There are two different kinds of sources which contribute to the uncertainty of the parameters:

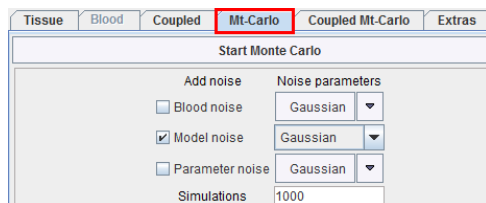
- 1) All the input data are measured and hence prone to measurement errors.

The model may contain more parameters than supported by the data, so that the effect of one parameter may be counter-balanced by another parameter.

The paradigm used for performing Monte Carlo simulations is described by Flannery et al. [11]. The basic idea is to simulate a series of measurements and statistically analyze the results when modeling these synthetic "measurements".

In PKIN Monte Carlo simulations can be performed for compartment models applied to a single tissue (**Mt-Carlo**) and models coupled across several tissues (**Coupled Mt-Carlo**) by selecting the respective panes.

### Single Tissue



Three types of "noise" can be defined. For each "noise" the distribution type as well as the variance has to be configured. After selecting the distribution a dialog window appears for defining the variance as described in the *Residual weighting* (on page 65) for model fitting.

- 1) **Blood noise** to be added to the blood data for simulating noisy input curves. This option is only supported for blood models of type **Lin. Interpolation**.

**Model noise** to be added to the tissue model curve for simulating TAC noise. Note that here only the noise distribution needs to be selected. The noise variance is adopted from the **Weighthing** tab on the **Tissue** pane.

**Parameter noise** to be added to the starting parameters for disturbing the starting conditions from the ideal solution. For instance 20% noise can be added to the ideal parameter values.

Three distributions are available for noise generation, **Gaussian**, **Poisson** and **Uniform**. They are applied for each individual measurement with the prescribed variance.

When the **Start Monte Carlo** button is selected the following processing starts:

- 1) An initial fit is performed. From then on it is assumed that the result parameters represent the "true" parameters, and the model curve represents the "true" measurement curve.

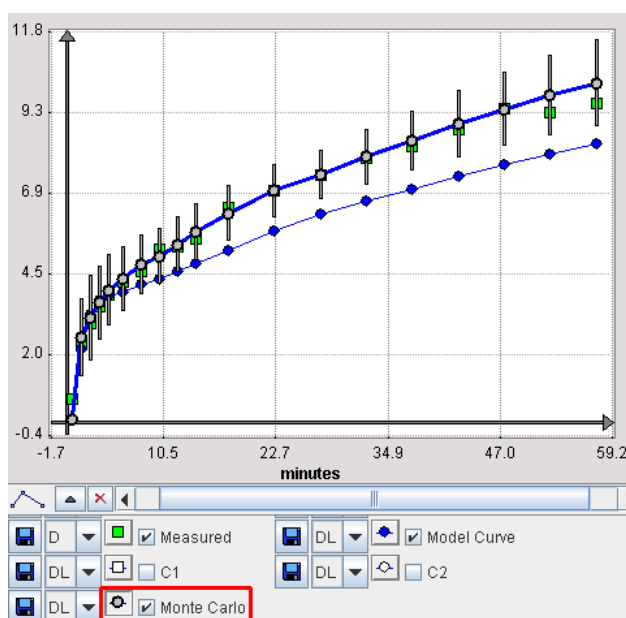
The number of **Simulations** times a noisy data set is prepared according to the definitions and then fitted. Noise is added to the "true" curve (and optionally the input curve). The "true" parameters, optionally disturbed by some noise, are used as starting values, and the fit to the noisy data performed. The resulting parameter values are recorded.

After all runs have completed, the distribution of the result parameters is analyzed resulting in a mean and a standard deviation value for each fitted parameter. These values are shown on the **Standard** tab of the **Tissue** page, and the model curve calculated with the mean parameters is shown. Often, it does not follow the measurement as well as with the fitted parameters. The Monte Carlo results should now be saved, because as soon as a parameter is changed or a new fit is initiated, the results get overwritten.

Parameter	Current value	Unit	% SE
<input type="checkbox"/> vB	0.05	1/1	---
<input checked="" type="checkbox"/> K1	0.067128	ml/ccm/min	2.16
<input checked="" type="checkbox"/> K2	0.127046	1/min	8.8
<input checked="" type="checkbox"/> K3	0.029199	1/min	42.95
<input checked="" type="checkbox"/> K4	0.050291	1/min	29.24
Vs	0.306778	ml/ccm	11.31
Vt	0.835155	ml/ccm	3.17
K1/K2	0.528377	ml/ccm	7.06
K3/K4	0.580605	1/1	17.02
Flux	0.012545	ml/ccm/min	---
ChiSquared	7.026279		

mean of MC  
Standard dev. of MC

It is possible to visualize a summary of the fit results during the Monte Carlo runs by switching back from the **Tissue** to the **Mt-Carlo** pane. In the curve display, a new curve **Monte Carlo** appears with vertical bars. The end points of these bars mark the minimal and the maximal value that any of the resulting model curves reached at a particular time point.



### Several Tissues Coupled

The same principle as for the single tissue Monte Carlo simulation applies to the Coupled Monte Carlo simulation (**Coupled Mt- Carlo** tab). In the upper section, the coupled parameters and the regions to couple are defined. In the lower section, blood and parameter noise can be specified. When starting, an initial coupled fit is performed to find the "true" parameters and the "true" model curves. Then, noisy curves are generated and coupled fits

performed. At the end, the mean and standard errors are available as the regional parameters.

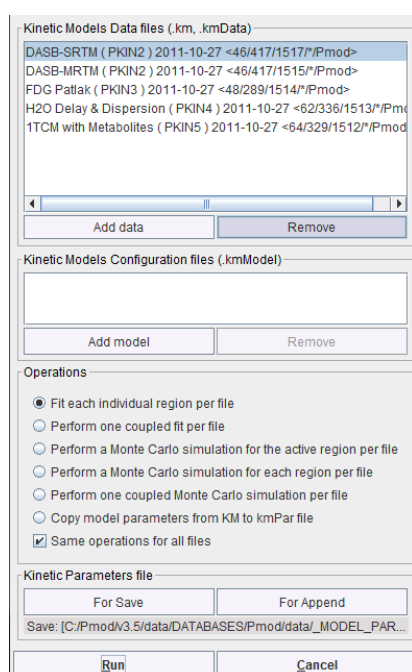
## Batch Mode

The **Batch Mode** serves for the sequential processing of multiple data sets. The processing, which can be applied to the data sets, depends on their format:

- ▶ **KM file:** A **.km** file saved from PKIN includes a complete definition of the blood interpolation model, the tissue model, weighting etc. for each region. Therefore, multiple processing types are supported including coupled fitting and Monte Carlo simulations. Additionally, during the fitting, the *options* (on page 63) on the **Extras** pane such as parameter initialization and randomized fitting will be effective. Note that the input file will be replaced by a new file with the results available in the *fitting history* (on page 68).
- ▶ **Composite data file:** A **.kmData** file contains only the *data definition* (on page 38), except for the global specification of a model. Therefore, only the individual tissue model fitting is supported. The result will be saved as a new .km file with the same name as the input file, and with the results in the history.

### Batch Facilities for .km Files

The **Batch Mode** entry from the **Kinetic** menu opens a dialog window for setting up the batch queue.



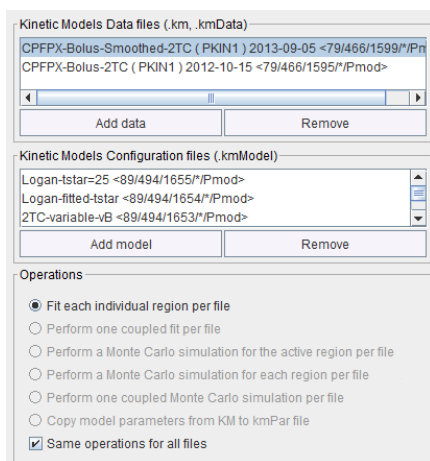
Use the **Add data** button for opening a data selection window and select the data sets to be processed from a database or the file system. Further data can incrementally be added, and selected list entries removed.

If only .km files have been selected it is assumed that appropriate configurations are included for the user's processing choice which is set by the radio button in the **Operations** section:

<b>Fit each individual region</b>	All regional TACs are fitted with the respective models as defined in the .km file.
<b>Perform one coupled fit</b>	Performs coupled fitting with the model and coupling as defined in the .km file.
<b>Perform a Monte Carlo simulation for the active region</b>	Using the noise and model definitions in the file a Monte Carlo simulation of the current TAC is performed.
<b>Perform a Monte Carlo simulation for each region</b>	As above, but applied to all regional TACs. Can be used to evaluate the sensitivity at different parameter combinations.
<b>Perform one coupled Monte Carlo simulation</b>	Monte Carlo simulation using coupled fitting as defined in the file.
<b>Copy model parameters from KM to kinPar file</b>	Just a convenience to summarize the parameters of a series of files into one tabular text file. Note that a result file must first be defined with the <b>For Save</b> button for the option to become available.

## Batch with Model Configuration Files

The batch mode also allows fitting multiple models to each data file. To do so, corresponding model configuration files can be selected with the **Add model** button.



In this situation, only the **Fit each individual region** option is available. The results of the different model fits are saved in the regional model histories.

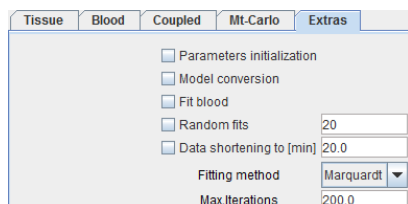
## Starting the Batch

The **For Save** and the **For Append** buttons can be used to specify a file for saving the result parameters (.kinPar) of the batch run.

The **Run** button starts processing, and PMOD will be blocked until processing completes.

## Fitting Blood Shape Parameters and Tissue Parameters together

The PKIN tool allows the simultaneous fitting of the shape parameters of the blood models and the kinetic parameters of the tissue model. This fitting mode can be activated by checking the **Fit blood** box on the **Extras** pane. It is active in the fitting of the single tissue TAC as well as in coupled fitting.



The screenshot shows the 'Extras' pane of the PKIN software. It contains several checkboxes and input fields. The 'Fit blood' checkbox is checked. Other checkboxes include 'Parameters initialization', 'Model conversion', and 'Random fits'. The 'Random fits' input field is set to 20. The 'Data shortening to [min]' input field is set to 20.0. The 'Fitting method' dropdown menu is set to 'Marquardt'. The 'Max iterations' input field is set to 200.0.

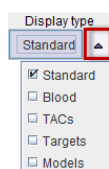
With **Fit blood** on, ALL parameters of the **Whole blood** and the **Plasma** model (but not the plasma fraction or parent fraction parameters) which have the fit check on will be modified during the fit of the kinetic tissue model. Note that the blood models are not fitted to the actual blood measurements in this process, but it is only the impact of the blood curve on the calculated tissue model curves which results in the modification of the blood model parameters.

## Convenience Tools

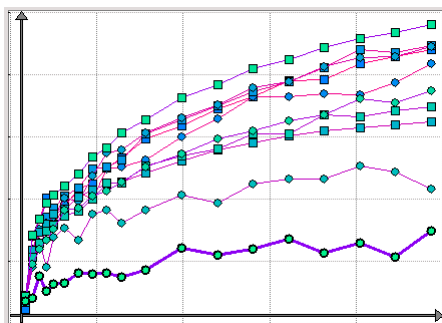
In the lower right part of PKIN there is an ensemble of user interface elements supporting some auxiliary tasks.

### Changing Display Types

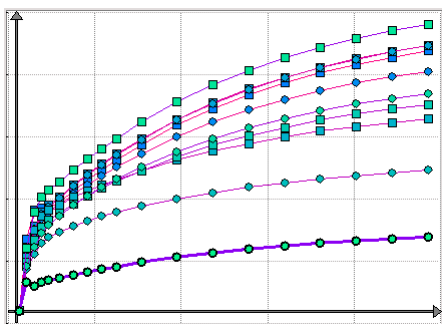
The **Display type** selection allows choosing which information is shown in the main curve area.



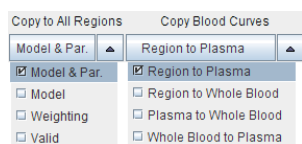
**Standard** shows all curves related to the current region. **Blood** shows only the blood measurements, **TACs** only the measured tissue data, **Targets** all regional target curves to be fitted,



and **Models** all fitted model curves.



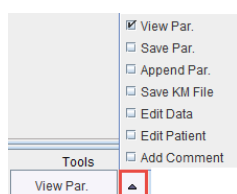
### Copy Operations



<b>Copy to all Regions</b>	<p>This multi-function button can be used to propagate a model from the current region to all regions in the study.</p> <p><b>Model &amp; Par.</b> copies the model with the parameters (which can then be used as starting values for <b>Fit all regions</b>).</p> <p><b>Model</b> copies the model structure, with the current parameters.</p> <p><b>Weighting</b> copies the current definition how the residuals are weighted.</p> <p><b>Valid</b> copies the definition of the currently valid points to all regional TACs.</p>
<b>Copy Blood Curves</b>	<p>This multi-function button allows transferring activity curves between the tissue and the blood.</p> <p><b>Region to Plasma</b> copies the current TAC to the input curve.</p> <p><b>Region to Whole Blood</b> copies the current TAC to the whole blood curve.</p> <p><b>Plasma to Whole Blood</b> copies the input curve to the whole blood curve.</p> <p><b>Whole Blood to Plasma</b> copies the whole blood curve to the input curve.</p>

## Auxiliary Tools

The **Tools** multi-function button is particularly helpful, if the PKIN tool is opened as an intermediate dialog and hence has no menu available. In this case, **Save KM**, **Save Par**, **Append Par** just allow to save the data as a .km file, as a parameter summary text file, and to append to a latter file.



**View Par** opens a dialog window and shows a quick summary of all regional model parameters as well as the statistical criteria, see *above* (on page 85).

**Edit Data** allows editing/processing the data values and generating new TACs, for instance by averaging TACs or adding noise, see *below* (on page 99).

**Edit Patient** allows changing the demographic patient data as well as the radionuclide, see *below* (on page 100).

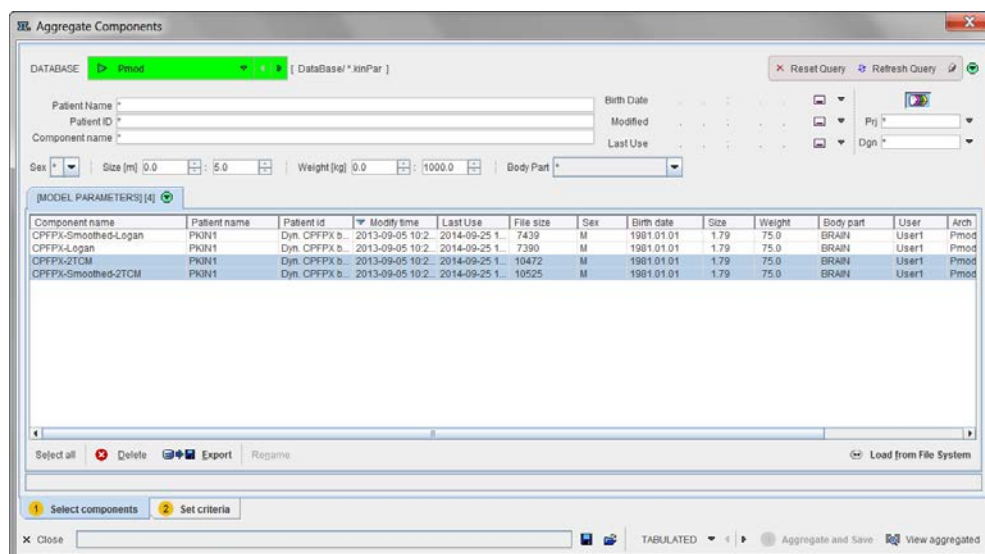
**Comments** opens a dialog window for entering arbitrary comments, see *below* (on page 101).

## Parameters Aggregation

The **Kinetic** menu contains an entry **Parameters Aggregation**. Its purpose is to concentrate the parameter values which have been saved in **KM Parameters Files (\*.kinPar)** for statistical analysis. The general concept and the details of data aggregation are explained in the *PMOD Base Functionality Guide*.

## Component Selection

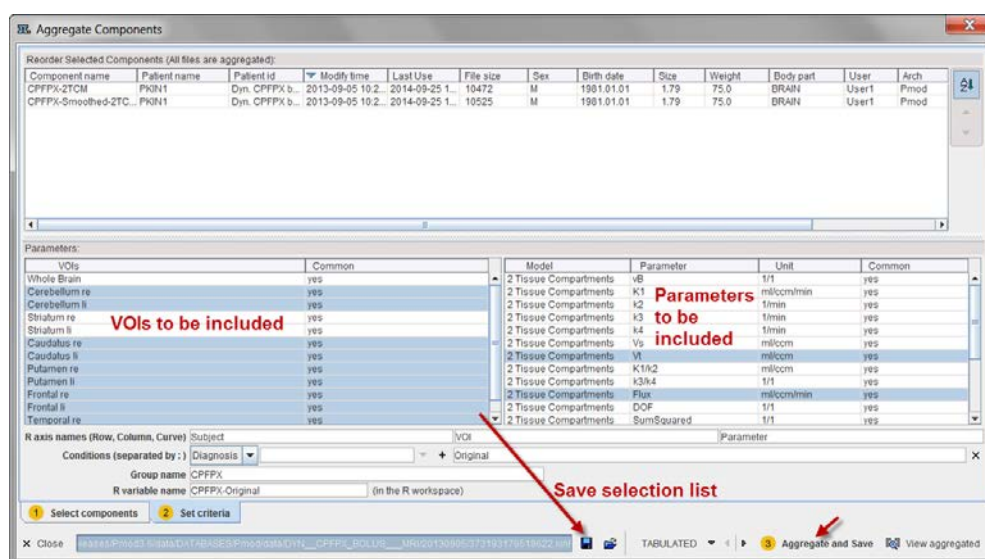
**Parameters Aggregation** opens a dialog window with two tabs as illustrated below. The **Select** pane allows defining the files from which the parameters are extracted.



They may be available in a database as in the example above, or in a disk directory. In the latter case use **Load from File System** to define the directory where the files reside. All appropriate **.kinPar** data sets are listed. Select all data sets to be used for the aggregation and then switch to the **Set Criteria** pane.

## Content Selection

The upper part **Reorder Selected Components** lists the data sets which have been selected for aggregation. They can be reordered if needed.



The **Parameters** section shows the available information. The left part lists the available VOIs, the right part the kinetic model parameters. A **yes** label in the **Common** columns indicates that all of the kinPar files include the particular element. Otherwise, a **no** label is



shown. Note that entries which are not available in a data set will result in a NaN value in the aggregate.

## Data Aggregation

Select the VOIs and parameters to be included so that they are highlighted, and activate **Aggregate and Save**. Specify the name of a database table, or a file name if the database is not used. When opening the saved aggregate with the **View aggregated** button, the statistics window appears with a selection for the **Statistics set**.

View Aggregate [2TC-AIIS]


Group 1 [Original: 2TC] X

Name PK3N1-5

ID Dyn\_CFPFX bolus & MRI-5

Study Date 20060301\_130226

☒ DP 6


 To Clipboard

CFPFX Bolus | Dynamic PET[Add\_Noise 0.5]

Statistics set 1 2TC-AIIS

VOI NAME	STATISTIC	VALUE	UNIT
Whole Brain	vB	0.05	1/1
	K1	0.067935	ml/ccm/min
	K2	0.126794	1/min
	K3	0.026616	1/min
	K4	0.049677	1/min
	Vs	0.289349	ml/ccm
Cerebellum re	Vt	0.829401	ml/ccm
	K1&2	0.540052	ml/ccm
	K3&4	0.535779	1/1
	vB	0.05	1/1
	K1	0.080625	ml/ccm/min
	K2	0.227533	1/min
Cerebellum li	K3	0.018952	1/min
	K4	0.039545	1/min
	Vs	0.169909	ml/ccm
	Vt	0.52425	ml/ccm
	K1&2	0.354342	ml/ccm
	K3&4	0.479505	1/1
Cerebellum li	vB	0.05	1/1
	K1	0.079229	ml/ccm/min
	K2	0.225351	1/min
	K3	0.018958	1/min
	K4	0.042339	1/min
	Vs	0.157509	ml/ccm
Vt	0.404009	ml/ccm	

X Close

 Add Statistics

## Application Cases

A typical example is the aggregation of the results of a test study in a test aggregate, and the results of the same population in a retest study in a retest aggregate. The two aggregates can then directly be used for a test-retest analysis using the PMOD R console.

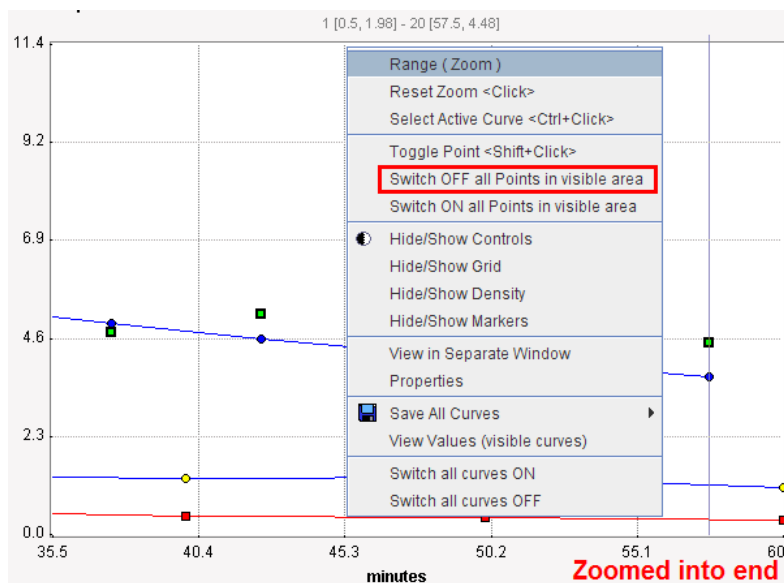
Another example is the comparison of two analysis methods. In this case the results of the different methods would be aggregated in corresponding aggregates, which can then be compared with a Bland-Altman analysis.

## Editing Facilities

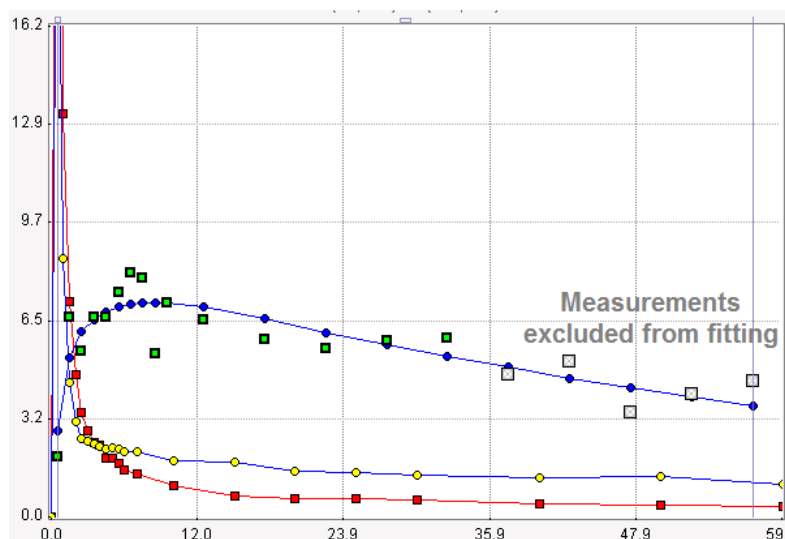
### Disable Measurements for Removing Outliers or Shortening the Fitted Data Segment

Occasionally there might be the need to disregard sample points when evaluating the matching criterion, for example to exclude an outlier, or to study the stability of model estimates with successive shortening of the acquisition duration.

In PKIN this can be easily achieved by (temporarily) disabling samples. First select the **Measured** curve by selecting it in the controls area or by **CTRL+Click** at a point. The green measurement points must appear highlighted in the display. Then individual points can be disabled by **SHIFT+Click** on them. For entire ranges it is advised to zoom into the region (drag left mouse button), then activate the context menu (right mouse button) and **Switch OFF all Points in visible area**. This operation is illustrated below



resulting in a "shortened" acquisition for fitting purposes:



Disabled measurements appear in grey. They can again be enabled by a second **SHIFT+Click**, or by **Switch ON all Points in visible area** in the context menu.

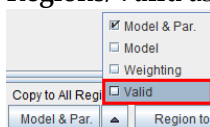
### Testing the Impact of Study Duration on Parameter Estimates

A frequent task is to find out the minimal study duration which allows the reliable estimation of a parameter. Such an analysis can be performed in PKIN as follows:

- 1) Ensure that the appropriate model has been configured for all regional TACs and **Fit all Regions**. Save the result parameters to a file using **Kinetic/Save KM Parameters File/Save**.

Disable the last valid measurement for a regional TAC.

Propagate this definition of valid points to all regional TACs using **Copy to All Regions/Valid** as illustrated below:



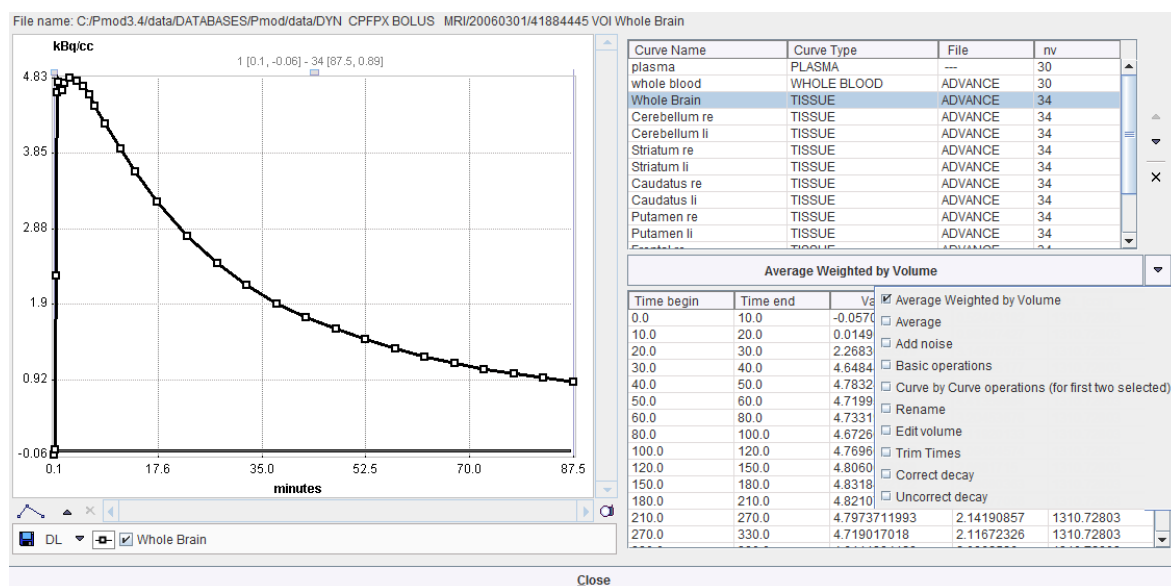
Fit again the model in all regions using the **Fit all Regions** button.

Add the parameter results from the shortened acquisition to the previous results using **Kinetic/Save KM Parameters File/Append**.

Repeat steps 2-5 continually shortening the acquisition until the parameter estimates degenerate. Then all data is available in a single file which can be opened in a program such as MS Excel for a statistical analysis.

## Edit Data and Create New Curves

The **Edit Data** button in the **Tools** list brings up a dialog window which allows to examine and manipulate the TAC data of the current data set.



The upper section lists the different curves. The blood-related curves are at the top, then come the regional TACs. The arrows to the right can be used to change the TAC ordering, and a TAC can be removed by the x button.

The lower section lists the data of the selected curve. Shown for each sample are the acquisition start and end times, the measurement value (usually the VOI average), its standard deviation (the VOI stdv, if available), and the VOI volume. Note that the volume may vary if the VOI definition changes throughout the time course. The values of the tissue TACs can be interactively edited by clicking into a cell and changing the number.

The configuration button between the lists houses the following operations to modify TACs and generate new TACs which will be appended to the list.

**Average Weighted by Volume** When several curves are selected, this button allows calculating the volume-weighted average TAC.

**Average** TAC averaging with equal weights.

**Add noise** Creates a copy of the selected TACs and adds noise according to the definition defined in a dialog window.

**Basic operations** Applies simple arithmetic operations to the values of the currently selected curves.

Add	0.0
Subtract	0.0
Multiply	1.0
Divide	1.0

**Curve by Curve operations** Arithmetic operations between the values of two selected TACs.

**Rename** For renaming of the currently selected curve.

**Edit volume** Change the volume of the selected TAC.

**Trim times** Check for frame time overlaps of the selected TACs and correct.

**Correct decay** Apply a decay correction to the selected TACs using the *isotope of the study* (on page 100).

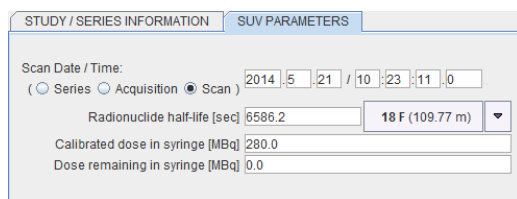
**Uncorrect decay** Undo the effect of decay correction for the selected TACs using the *isotope of the study* (on page 100).

## Edit Patient and Change Isotope

The **Edit Patient** button in the **Tools** list brings up a dialog window which allows you to examine and change the demographic information of the current data set.

In case the TAC data was transferred from PVIEW to PKIN, this information is automatically filled in from the image data (if available in the data format). However, the contents can also be edited and saved if required. If the patient is already in the database, the **IMPORT PATIENT INFO FROM** facility can be used. With the **DATABASE** button a selection window opens which allows choosing an image series from the database. The required information will be retrieved and filled into the text fields.

Note the **SUV PARAMETERS** tab which gives access to the isotope definition.



STUDY / SERIES INFORMATION    SUV PARAMETERS

Scan Date / Time:  
 ( ☐ Series ☐ Acquisition ☒ Scan ) 2014 . 5 . 21 / 10 : 23 : 11 . 0

Radionuclide half-life [sec] 6586.2    18 F (109.77 m) ▼

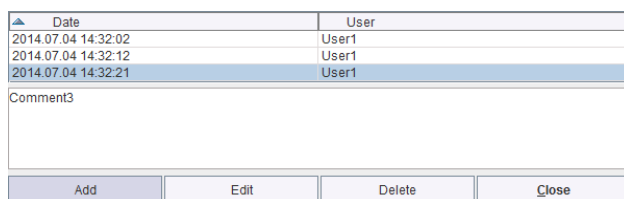
Calibrated dose in syringe [MBq] 280.0

Dose remaining in syringe [MBq] 0.0

This information is relevant for some models as well as calculated *residual weighting* (on page 65).

## Comments

The **Add Comment** button in the **Tools** list allows incrementally adding descriptive text to the data set being processed. It opens a dialog window with two areas: a list of comment entries with date and name of the creator in the upper part, and the actual text of the selected comment entry in the lower part.



Date	User
2014.07.04 14:32:02	User1
2014.07.04 14:32:12	User1
2014.07.04 14:32:21	User1

Comment3

Add    Edit    Delete    Close

To create a new comment entry select **Add**, type the comment into the lower text window, and complete editing by **Set**. This operation creates a new list entry which can be selected to show its contents, changed with the **Edit** button, or removed with **Delete**.

Note that you have to use **Save KM File** to include updated comments permanently into the data set.

## Synthetic Data Generation

The compartment models in PKIN can be employed for creating data sets representing a perfect measurements with a certain acquisition protocol of the tissue and blood activities.

### Synthetic Modeling Study for PKIN

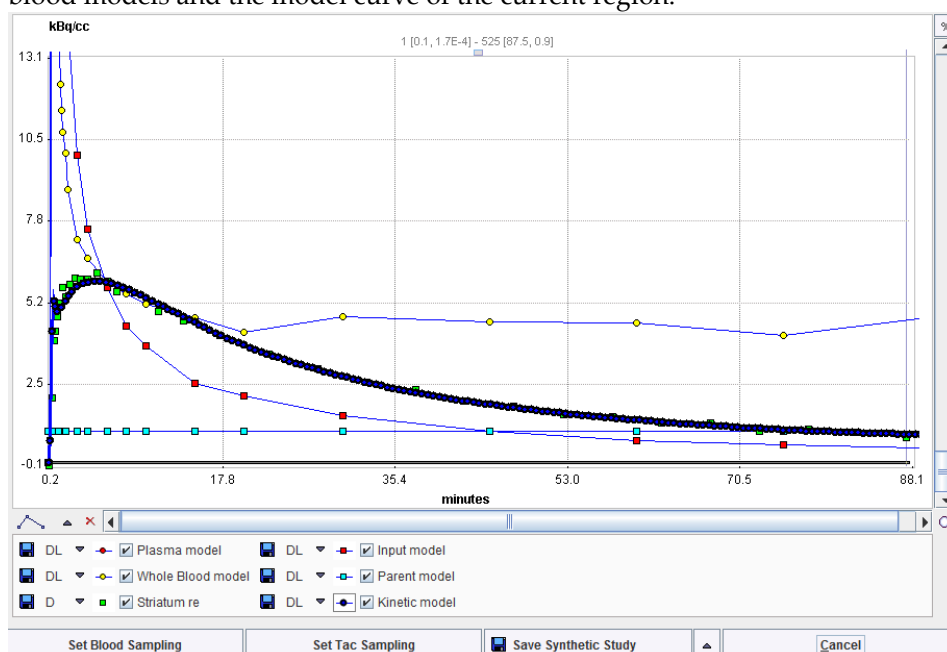
PKIN always shows the model curve given the loaded input curve(s) and the configured regional kinetic model with the timing of the current study. The user may modify any of the defining parameters and observe the effect on the model curve, which represents the expected PET measurement. With the **Create Synthetic KM Study** facility well-defined and smooth TACs and blood curves can be generated which represent an ideal acquisition.

#### Synthetic Modeling Study Generation

1) Define a suitable kinetic model (compartment model, parameter values) in all regions.

Define a suitable input curve model, for instance a smooth curve fitted to the actual input curve.

Select the entry **Synthetic KM Study** in the **Kinetic** menu. A dialog appears showing the blood models and the model curve of the current region.



Initially all curves have the original sampling times. To modify sampling, use the buttons **Set Blood Sampling** and **Set Tac Sampling**. For instance, very fine sampling allows to generate smooth curves for publication or simulation purposes.

When **Save Synthetic Study** is activated, all regional TACs are recalculated with the current sampling, the old measurements are replaced by the values of the model curve, the original blood values are replaced by the values calculated with the current blood models, and the data is saved in a file specified by the user. Then, the original data are shown again. As a convenience for adding the generated data set there is the button

Save & Append as new Study available underneath:



Finally, to see and work with the synthetic study, load the created file. The synthetic model curves are now the measured TACs, and the resampled blood curves have the **Lin. Interpolation** model associated. So any underlying model of the blood curve (which has been used for synthetic model calculation) gets lost. This may cause small deviations between the calculated model curve and the saved synthetic TAC, depending on the density of blood sampling.

## Synthetic Imaging Study for PXMOD Tests

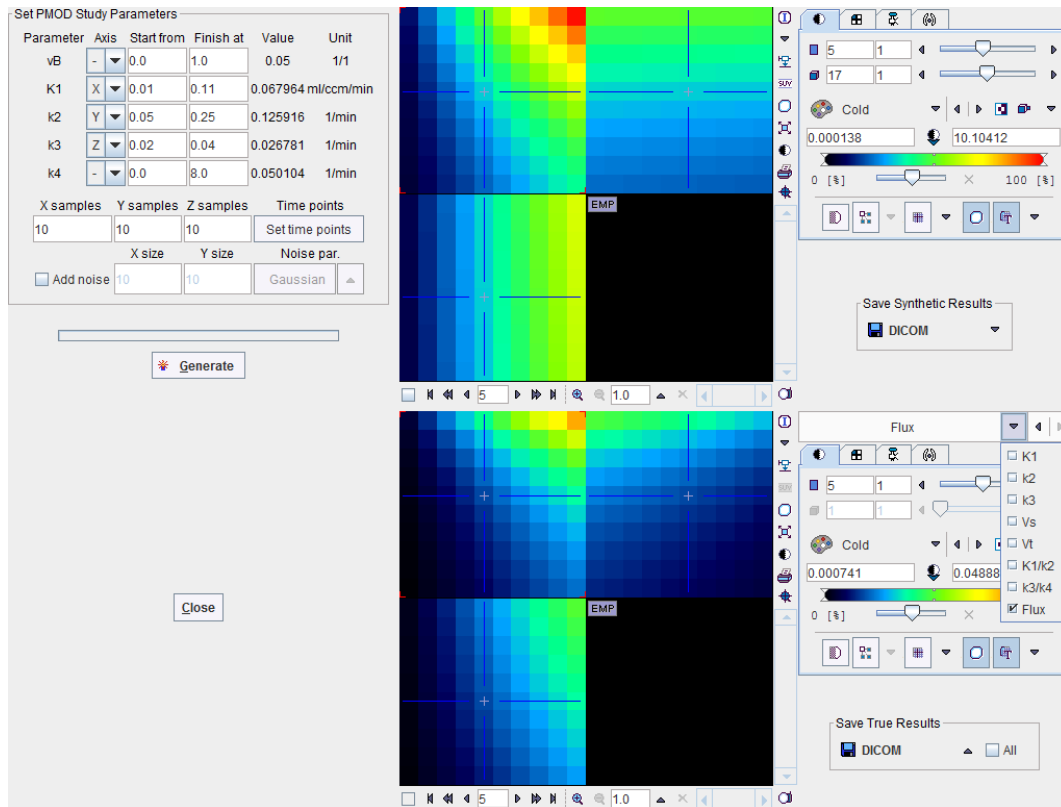
**Create Synthetic PXMOD Study** is an extension of the **Create Synthetic KM Study** approach. The idea is that synthetic model curves can be generated for many different "tissues", ie. combinations of model parameters, at once. These curves are compiled into images, so that a synthetic study is generated which has a different combination of model parameters in each pixel.

For instance, TACs can be simulated by loading a .km file, selecting the 2-tissue compartment model and using the parameter ranges

Parameter	From	To
$K_1$	0.05	0.1
$k_2$	0.05	0.25
$k_3$	0.02	0.04
$k_4$	fixed to 0.05	



To enter these parameter ranges select **Create Synthetic PMXOD Study** from the **File** menu. The following dialog appears showing the parameters of the current model.



To define the synthetic study the following configurations must be set:

- ▶ Which parameters are varied along the **X**, **Y**, and **Z** axis (**Axis** selection),
- ▶ the value ranges of the varied parameters (**Start from**, **Finish at**),
- ▶ the number of parameter increments to cover the ranges (**X**, **Y**, **Z** **samples**),
- ▶ the acquisition times; initially the times of the loaded .km study are used, but they can be changed using the **Set time points**.

With **Generate** the model curves of all parameter combinations are generated and the data arranged as a 4-dimensional study. This synthetic study, which is suitable for pixel-wise modeling, is shown in the upper image display. In the example above the interpolation has been disabled to show the 10x10 layout of parameter combinations in plane. Use the **Save Synthetic Results** button to save the study in any supported output format.

The lower image display contains the true parameter images. These generation parameters can be saved using the **Save True Results** button. They can serve as the reference for the analysis of the outcome from a pixel-wise analysis of the synthetic study.

## Modeling in Batch Pipeline

The PKIN models can be applied as part of a processing pipeline. For instance, a dynamic PET series can be motion corrected, an atlas fitted to the data and the regional TACs calculated which are then fitted by a model.

### Association of Blood Components to Dynamic PET

For blood-based models the related information such as input curve, whole blood curve, and the parent fraction of an dynamic PET must be specified. This requires that the data resides in a PMOD database and uses the "association" concept which allows defining relationships between data elements.

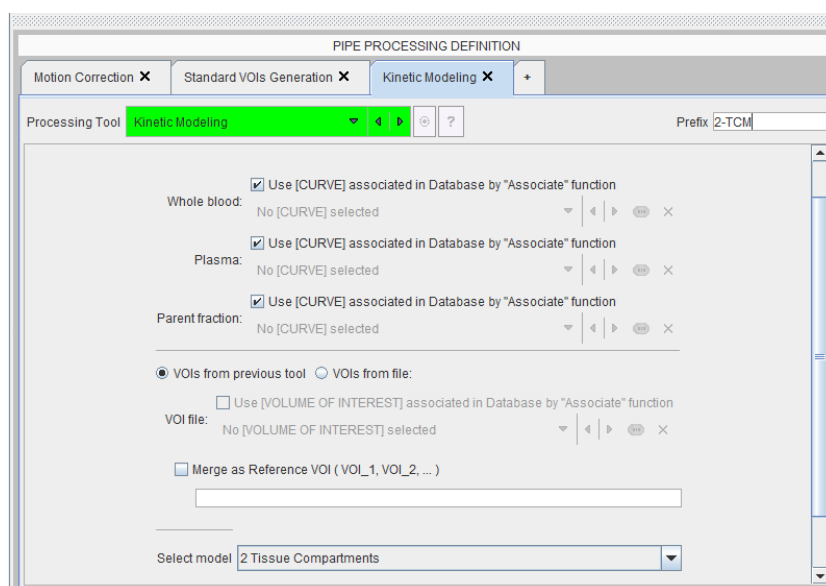
For instance, the whole-blood curve is specified as follows. In the **Patients** list of the database browser select the patient, and in the **Series** list the dynamic PET. Next, activate the association list as illustrated below and select the **Associate Whole blood** entry.

The screenshot displays the PMOD database browser interface. The top section, 'Patients [23]', contains a table with columns: Patient name, Patient ID, Modify date, Sex, and Date of Birth. Patient PKIN2 is highlighted. Below this is a toolbar with buttons like 'Set as "Selected for Loading"', 'Edit Patient', 'Delete Patient(s)', 'Create new Patient', 'Set Project', and 'Merge'. The bottom section, 'Series [2]', contains a table with columns: Patient Name, Study date, Time, Study description, Series description, Modified, Last Use, Mod, nz, nv, and nd. The 'Dynamic DASB PET' series for PKIN2 is highlighted. Below the series list is another toolbar with buttons like 'Add', 'Add All', 'Edit', 'Delete', 'Associate Whole blood', and a dropdown menu. The dropdown menu is open, showing options: 'Set Project', 'Associate Images', 'Associate VOI', 'Associate Whole blood' (which is checked), 'Associate Plasma', and 'Associate Parent fraction'.

A component browser appears which is used to select the whole-blood curve. The same procedure is repeated for the input curve with the **Associate Plasma** entry, and if applicable with the **Associate Parent fraction**. From then on the roles of the blood curves is established, and selection of the dynamic PET is sufficient for modeling purposes.

## Kinetic Modeling External Tool

Please refer to the *PMOD Base Tool Users Guide* for general explanations about setting up batch pipe processing. A modeling stage can be included by means of the **Kinetic Modeling** external tool which has the interface illustrated below.



For blood-based models the whole-blood, plasma and parent fractions have to be specified. Unless data of the same study is processed multiple times with the same blood data, the associations have to be used for a general specification. With **VOIs from previous tool** the VOI definitions produced by the pipeline will be applied. Again, **VOIs from file** can only be applied when processing the same data multiple times, or if the images have been normalized to a standard space.

For reference tissue models the reference TAC can be generated by combining a set of VOIs. The set has to be specified in a comma-separated list such as **Cerebellum\_1,Cerebellum\_r**, or using wildcards such as **Cerebellum\***.

Finally, the tissue model to be fitted to each regional TAC can be chosen from the **Select model** list.

## Chapter 3

# PKIN Model Reference

PKIN implements different types of models with somewhat different properties.

- 1) Compartment Models require an input curve for the calculation of the expected concentrations in the different compartments. Most of them also support a blood spillover correction term.

Reference models do not use explicit blood information. Rather, a reference TAC is specified which must satisfy specific criteria. The reference TAC is used to calculate the response in the tissue of interest.

Non-compartment models perform different types of analyses which are not based on a compartment structure of the model. Most of them use the input curve.

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**Note:** It is assumed that all data used has been **decay corrected** to the same time point.

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# Compartment Models

## List of PKIN Compartment Models

PKIN features a comprehensive set of compartment models as listed below. The number of tissue compartments ranges from 1 to 3. There are different variants of the same model structure, so that prior information can be entered easily, and that coupling of physiologic parameters across region is possible.

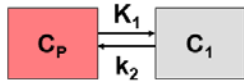
For instance, the 2-tissue compartment model has the standard parameters  $K_1$ ,  $k_2$ ,  $k_3$ ,  $k_4$ . An equivalent description is by the parameters  $K_1$ ,  $K_1/k_2$ ,  $k_3$ ,  $k_4$ . The advantage is, that the  $K_1/k_2$  represents the distribution volume of the non-displacement compartment in tissue (free and non-specifically bound tracer), which can often be assumed to be the same across different tissues. Therefore,  $K_1/k_2$  can be included as a common parameter in a coupled fit, hereby reducing the number of fitted parameters and thus potentially improving the identifiability of all.

Model Name	Description
<i>1-Tissue Compartment</i> (on page 110)	Most basic compartment model with the plasma compartment and one tissue compartment.
<i>2-Tissue Compartments</i> (on page 110)	Compartment model with the plasma compartment and two sequential tissue compartments. Often used for receptor studies.
<i>FDG-2 Tissue Compartments</i> (on page 110)	The 2-Tissue compartment model including the plasma glucose and the lumped constant to calculate the metabolic rate of glucose.
<i>2-Tissue Compartments, <math>K_1/k_2</math></i> (on page 110)	The same model as the 2-Tissue compartment model, except that $K_1/k_2$ is used as a model parameter instead of $k_2$ . This facilitates coupled fitting.
<i>2-Tissue Compartments, <math>K_1/k_2</math> &amp; <math>V_s</math></i> (on page 110) <i>2-Tissue Compartments, <math>K_1/k_2</math> &amp; <math>V_t</math></i> (on page 112)	The same models as the 2-Tissue compartment model, except that $K_1/k_2$ and the specific or total distribution volumes are used as model parameters instead of $k_2$ and $k_4$ . This facilitates coupled fitting and the easy generation of synthetic model curves.
<i>Linear Least Squares</i> (on page 114)	2-Tissue Compartment model solved by the Linear Least Squares method.
<i>2-Tissue Compartments, <math>k_5</math></i> (on page 116)	2-Tissue compartment model with an efflux from the last compartment.
<i>2-Tissue Compartments, <math>B_{max}</math></i> (on	Non-linear 2-Tissue compartment model for

page 116)	receptor tracer studies accounting for the saturation of receptor sites.
<b>2-Tissue Compartments, <math>K_1/k_2</math>, <math>B_{max}</math></b> (on page 116)	Same model as above, but using $K_1/k_2$ as fitting parameter instead of $k_2$ .
<b>2-Tissue Compartments, vascular trapping</b> (on page 118)	2-Tissue compartment model with an additional endothelial trapping compartment.
<b>3-Tissue Compartments</b> (on page 119)	Compartment model which separates free tracer in tissue from non-specific binding.
<b>3-Tissue Compartments, <math>K_1/k_2</math>, <math>V_s</math></b> (on page 120)	As above, but using but using $K_1/k_2$ and the specific distribution volume as fitting parameters.
<b>3-Tissue Compartments, sequential</b> (on page 120)	Model with three sequential compartments originally developed for FDG uptake in skeletal muscle.
<b>Flow &amp; Dispersion</b> (on page 127)	Specific model for dynamic $H_2^{15}O$ - PET Data with implicit deconvolution of the input curve dispersion
<b>3-Tissue Compartments, Metabolites</b> (on page 121)	Extends the 2 tissue compartment model by a metabolite compartment with a second input curve of labeled metabolites from the plasma.
<b>3-Tissue Compartments, Metabolites, <math>K_1/k_2</math></b> (on page 122)  <b>3-Tissue Compartments, Metabolites, <math>K_1/k_2</math>, <math>V_s</math></b> (on page 123)	As above, but using but using $K_1/k_2$ and the specific distribution volume as fitting parameters.
<b>4-Tissue Compartments, Metabolites, <math>K_1/k_2</math></b> (on page 124)	Two parallel 2-compartment models for authentic ligand and metabolites, linked by a transfer constant between the non-specific compartments.
<b>Triple-injection Protocol for Flumazenil</b> (on page 125)	During a single imaging study three injections are applied: hot ligand first, then cold ligand for displacement, then a mixture of cold & hot ligand. The individual receptor parameters can be estimated.

## 1-Tissue Compartment Model

The 1-Tissue Compartment Model only separates tissue uptake and washout. All tracer in tissue is included in a single compartment  $C_1$ .



### Differential Equation of the Mass Balance

$$\frac{dC_1(t)}{dt} = K_1 C_P(t) - k_2 C_1(t)$$

with input curve  $C_P(t)$ , tissue concentration  $C_1(t)$ , uptake rate constant  $K_1$  [ml/ccm/min] and clearance rate constant  $k_2$  [1/min].

### Operational Model Curve

$$C_{Model}(t) = (1 - v_B) C_1(t) + v_B C_B(t)$$

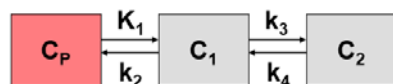
with the concentration  $C_B(t)$  of tracer in whole blood, and the blood volume fraction  $v_B$ .

### Parameter Fitting

The model includes the three fitable parameters  $v_B$ ,  $K_1$ ,  $k_2$ . Often,  $v_B$  is fixed at a physiologic value of 3-5%.

## 2-Tissue Compartment Model

The 2-tissue compartment model separates tracer in tissue into two exchanging compartments  $C_1$  and  $C_2$  with a serial structure. The arterial plasma exchanges with the first tissue compartment  $C_1$ , which in turn exchanges with the second tissue compartment  $C_2$ .  $K_1$  [ml/ccm/min] and  $k_2$  [1/min] are the uptake and clearance rate constants, whereas  $k_3$  [1/min] and  $k_4$  [1/min] describe the exchange between the tissue compartments. The usual interpretation is that  $C_1$  represents free and non-specifically bound tracer in tissue (non-displaceable compartment), and  $C_2$  specifically bound tracer.



### Differential Equation of the Mass Balance

$$\begin{aligned} \frac{dC_1(t)}{dt} &= K_1 C_P(t) - (k_2 + k_3) C_1(t) + k_4 C_2(t) \\ \frac{dC_2(t)}{dt} &= k_3 C_1(t) - k_4 C_2(t) \end{aligned}$$

with input curve  $C_P(t)$ .

## Operational Model Curve

$$C_{Model}(t) = (1 - v_B)(C_1(t) + C_2(t)) + v_B C_B(t)$$

with the concentration  $C_B(t)$  of tracer in whole blood, and the blood volume fraction  $v_B$ .

## Parameter Fitting

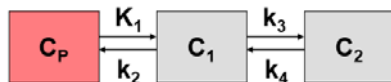
The model includes the 5 fitable parameters **vB**, **K1**, **k2**, **k3**, **k4**. Usually,  $v_B$  is fixed at a physiologic value of 3-5% to reduce the number of fitted parameters. An alternative is to fit  $v_B$  with a TAC from a big representative VOI, and fix it for the TACs from smaller VOIs. For irreversible tracers with trapping in the second compartment,  $k_4$  is fixed: **k4=0**. Please inspect the %SE standard error to get information about the reliability of the parameter estimates.

## Alternative Models with VND and VS Fit Parameters

There are two alternative models available which have the same structure but use distribution volumes as fit parameters. The advantage of the reformulation is that in some situations distribution volumes can be assumed to be common among regions and estimated by a coupled fit, or they might already be known and can thus be fixed at a prior value.

## 2-Tissue Compartment Model, K1/k2

The 2-tissue compartment model separates tracer in tissue into two exchanging compartments  $C_1$  and  $C_2$  with a serial structure. The arterial plasma exchanges with the first tissue compartment  $C_1$ , which in turn exchanges with the second tissue compartment  $C_2$ .  $K_1$  [ml/ccm/min] and  $k_2$  [1/min] are the uptake and clearance rate constants, whereas  $k_3$  [1/min] and  $k_4$  [1/min] describe the exchange between the tissue compartments. The usual interpretation is that  $C_1$  represents free and non-specifically bound tracer in tissue (non-displaceable compartment), and  $C_2$  specifically bound tracer.



## Differential Equation of the Mass Balance

$$\begin{aligned} \frac{dC_1(t)}{dt} &= K_1 C_p(t) - (k_2 + k_3) C_1(t) + k_4 C_2(t) \\ \frac{dC_2(t)}{dt} &= k_3 C_1(t) - k_4 C_2(t) \end{aligned}$$

with input curve  $C_p(t)$ .

## Operational Model Curve

$$C_{Model}(t) = (1 - v_B)(C_1(t) + C_2(t)) + v_B C_B(t)$$

with the concentration  $C_B(t)$  of tracer in whole blood, and the blood volume fraction  $v_B$ .



### Parameter Fitting using $K_1/k_2$ instead of $k_2$

The **2-Tissue Compartment Model,  $K_1/k_2$**  model variant uses  $K_1/k_2$  as a fitting parameter instead of  $k_2$ .  $K_1/k_2$  represents the distribution volume  $V_{ND}$  of the non-displaceable compartment  $C_1$ . The advantage of this formulation is that in some situations  $V_{ND}$  can be assumed to be common among regions and estimated by a coupled fit. Alternatively,  $V_{ND}$  might already be known and can thus be fixed at a prior value. The other four fitting parameters  $v_B$ ,  $K_1$ ,  $k_3$ ,  $k_4$  are the same as for the **2-Tissue Compartment Model**.

## 2-Tissue Compartment Model, $K_1/k_2$ , $V_s$

The 2-tissue compartment model separates tracer in tissue into two exchanging compartments  $C_1$  and  $C_2$  with a serial structure. The arterial plasma exchanges with the first tissue compartment  $C_1$ , which in turn exchanges with the second tissue compartment  $C_2$ .  $K_1$  [ml/ccm/min] and  $k_2$  [1/min] are the uptake and clearance rate constants, whereas  $k_3$  [1/min] and  $k_4$  [1/min] describe the exchange between the tissue compartments. The usual interpretation is that  $C_1$  represents free and non-specifically bound tracer in tissue (non-displaceable compartment), and  $C_2$  specifically bound tracer.



### Differential Equation of the Mass Balance

$$\begin{aligned} \frac{dC_1(t)}{dt} &= K_1 C_p(t) - (k_2 + k_3) C_1(t) + k_4 C_2(t) \\ \frac{dC_2(t)}{dt} &= k_3 C_1(t) - k_4 C_2(t) \end{aligned}$$

with input curve  $C_p(t)$ .

### Operational Model Curve

$$C_{Model}(t) = (1 - v_B) (C_1(t) + C_2(t)) + v_B C_B(t)$$

with the concentration  $C_B(t)$  of tracer in whole blood, and the blood volume fraction  $v_B$ .

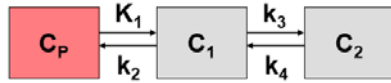
### Parameter Fitting using $K_1/k_2$ and $V_s$

This **2-Tissue Compartment Model,  $K_1/k_2$ ,  $V_s$**  model variant uses  $K_1/k_2$  and  $V_s$  as fitting parameters instead of  $k_2$  and  $k_4$ .  $V_s (=K_1/k_2 * k_3/k_4)$  represents the distribution volume of the specific compartment  $C_2$ . This formulation has particular advantages for the generation of synthetic data with predefined distribution volumes. The other three fitting parameters  $v_B$ ,  $K_1$ , and  $k_3$  are the same as for the **2-Tissue Compartment Model**.

## 2-Tissue Compartment Model, $K_1/k_2$ , $V_t$

The 2-tissue compartment model separates tracer in tissue into two exchanging compartments  $C_1$  and  $C_2$  with a serial structure. The arterial plasma exchanges with the first

tissue compartment  $C_1$ , which in turn exchanges with the second tissue compartment  $C_2$ .  $K_1$  [ml/ccm/min] and  $k_2$  [1/min] are the uptake and clearance rate constants, whereas  $k_3$  [1/min] and  $k_4$  [1/min] describe the exchange between the tissue compartments. The usual interpretation is that  $C_1$  represents free and non-specifically bound tracer in tissue (non-displaceable compartment), and  $C_2$  specifically bound tracer.



### Differential Equation of the Mass Balance

$$\begin{aligned}\frac{dC_1(t)}{dt} &= K_1 C_P(t) - (k_2 + k_3) C_1(t) + k_4 C_2(t) \\ \frac{dC_2(t)}{dt} &= k_3 C_1(t) - k_4 C_2(t)\end{aligned}$$

with input curve  $C_P(t)$ .

### Operational Model Curve

$$C_{Model}(t) = (1 - v_B)(C_1(t) + C_2(t)) + v_B C_B(t)$$

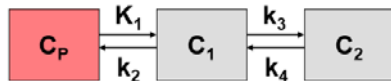
with the concentration  $C_B(t)$  of tracer in whole blood, and the blood volume fraction  $v_B$ .

### Parameter Fitting using $K_1/k_2$ and $V_t$

This **2-Tissue Compartment Model,  $K_1/k_2$ ,  $V_t$**  model variant uses  $K_1/k_2$  and  $V_t$  as fitting parameters instead of  $k_2$  and  $k_4$ .  $V_t (=K_1/k_2 * (1+k_3/k_4))$  represents the total distribution volume of tracer in tissue. This formulation has particular advantages for the generation of synthetic data with predefined distribution volumes. The other three fitting parameters  $v_B$ ,  $K_1$ , and  $k_4$  are the same as for the **2-Tissue Compartment Model**.

## FDG 2-Tissue Compartment Model

The 2-tissue compartment model for FDG has the same structure as the standard model.



### Differential Equation of the Mass Balance

$$\begin{aligned}\frac{dC_1(t)}{dt} &= K_1 C_P(t) - (k_2 + k_3) C_1(t) + k_4 C_2(t) \\ \frac{dC_2(t)}{dt} &= k_3 C_1(t) - k_4 C_2(t)\end{aligned}$$

with FDG concentration of in arterial plasma  $C_P(t)$ , FDG concentration  $C_1(t)$  and phosphorylated FDG (FGD-6-P) concentration  $C_2(t)$  in tissue.  $K_1$  [ml/ccm/min] and  $k_2$  [1/min] are the uptake and clearance rate constants, whereas  $k_3$  [1/min] represents phosphorylation by hexokinase and  $k_4$  [1/min] dephosphorylation.

## Operational Model Curve

$$C_{Model}(t) = (1 - v_B)(C_1(t) + C_2(t)) + v_B C_B(t)$$

with the concentration  $C_B(t)$  of tracer in whole blood, and the blood volume fraction  $v_B$ .

## Parameter Fitting

The model includes the 5 fitable parameters **vB**, **K1**, **k2**, **k3**, **k4**. Usually, **vB** is fixed at a physiologic value of 3-5% to reduce the number of fitted parameters, and dephosphorylation is so slow that  $k_4$  is fixed **k4=0**.

## FDG-specific Results

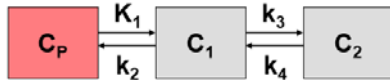
The FDG model has two additional input parameters, the lumped constant (**LC**) and the plasma glucose concentration (**PG** [mmol/l]). In combination with the estimated  $K_1$ ,  $k_2$ , and  $k_3$  parameters they allow calculating the metabolic rate of glucose MRGlu

$$MRGlu = \frac{PG}{LC} \frac{K_1 k_3}{k_2 + k_3}$$

Note that the **LC** and **PG** values are maintained when switching back and forth with the **Patlak (on page 163)** model, if the **Model conversion** option in the **Extras** panel is enabled.

## 2-Tissue Compartment Model, Linear Least Squares Method

In addition to uptake and washout the model handles two exchanging compartments  $C_1$  and  $C_2$  in tissue.



## Differential Equation of the Mass Balance

$$\begin{aligned} \frac{dC_1(t)}{dt} &= K_1 C_P(t) - (k_2 + k_3) C_1(t) + k_4 C_2(t) \\ \frac{dC_2(t)}{dt} &= k_3 C_1(t) - k_4 C_2(t) \end{aligned}$$

with input curve  $C_P(t)$ .

## Operational Model Curve

With linearized solutions, the differential equations are integrated twice on both sides, substitutions performed and finally rearranged. This can be done in different ways. The current model uses the derivation of Cai et al. [1], equation (6):

$$C_{Model}(t) = P_1 C_B(t) + P_2 \int_0^t C_P(\tau) d\tau + P_3 \int_0^t \int_0^\tau C_P(s) ds d\tau + P_4 \int_0^t C_{PET}(\tau) d\tau + P_5 \int_0^t \int_0^\tau C_{PET}(s) ds d\tau$$

with input curve  $C_p(t)$ , the concentration  $C_b(t)$  of tracer in whole blood, and the tissue curve to be fitted  $C_{PET}(t)$ .

### Parameter Fitting

The multi-linear operational equation can be solved in a least squares sense in one step using a singular value decomposition method. From the resulting 5 coefficients  $P_1, \dots, P_5$  the target parameters can be calculated as follows:

$$\begin{aligned} vB &= P_1 & k_3 &= -(k_2 + k_4 + P_4) \\ K_1 &= \frac{P_1 P_4 + P_2}{1 - P_1} & k_4 &= \frac{-P_3}{k_2} \\ k_2 &= -\frac{P_1 P_3 + P_3}{P_1 P_4 + P_2} - P_4 \end{aligned}$$

### Recommended Use of the Linear Least Squares Method

The advantages of the linearized approach are fast computation, a well-defined solution, and no problem with local minima. However, it is well known that the method is susceptible to bias, and that small TAC perturbations can cause large changes of the parameter estimates. Therefore it is recommended that the **Linear Least Squares** model is only used for getting a quick solution which is further refined by the iterative methods. In fact, if **Parameters Initialization** option is configured in the **Extras** panel, the initial starting parameters of the compartment models are obtained by the **Linear Least Squares** method.

### Implementation Notes

The model works as follows for different model configurations:

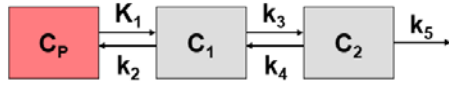
- ▶▶ Irreversible model: if **k4** is disabled from fitting, the program automatically sets **k4** = 0 and uses a linearized equation which is modified accordingly.
- ▶▶ 1-tissue compartment model: if **k3** is disabled from fitting, the program automatically sets **k3** = **k4** = 0 and uses a modified equation.
- ▶▶ In all configurations the **vB** parameter can optionally be disabled from fitting and fixed at a specific user-defined value.

### Reference

1. Cai W, Feng D, Fulton R, Siu WC: Generalized linear least squares algorithms for modeling glucose metabolism in the human brain with corrections for vascular effects. *Comput Methods Programs Biomed* 2002, 68(1):1-14.

## 2-Tissue Compartment Model, k5

In addition to uptake and washout the model handles two exchanging compartments  $C_1$  and  $C_2$  in tissue, as well as an efflux from  $C_2$ .



### Differential Equation of the Mass Balance

$$\begin{aligned}\frac{dC_1(t)}{dt} &= K_1 C_p(t) - (k_2 + k_3) C_1(t) + k_4 C_2(t) \\ \frac{dC_2(t)}{dt} &= k_3 C_1(t) - (k_4 + k_5) C_2(t)\end{aligned}$$

with input curve  $C_p(t)$ , concentration  $C_1(t)$  in the first and concentration  $C_2(t)$  in the second compartment.  $K_1$  [ml/ccm/min] and  $k_2$  [1/min] are the uptake and clearance rate constants, whereas  $k_3$  [1/min] and  $k_4$  [1/min] describe the exchange between the tissue compartments. The typical interpretation is that  $C_1$  represents free and non-specifically bound tracer in tissue (non-displaceable compartment), and  $C_2$  represents specifically bound tracer.

### Operational Model Curve

$$C_{Model}(t) = (1 - v_B)(C_1(t) + C_2(t)) + v_B C_B(t)$$

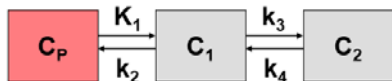
with the concentration  $C_B(t)$  of tracer in whole blood, and the blood volume fraction  $v_B$ .

### Parameter Fitting

The model includes the 6 fitable parameters **vB**, **K1**, **k2**, **k3**, **k4**, **k5**. Usually, **vB** is fixed at a physiologic value of 3-5% to reduce the number of fitted parameters. Per default, **k4=0** is fixed to model an efflux from the specific compartment rather than an exchange with the non-displaceable compartment.

## 2-Tissue Compartment Model, Bmax

The saturable receptor-ligand model includes two tissue compartments.  $C_1$  represents the non-displaceable tissue compartment with free and non-specifically bound tracer, and  $C_2$  tracer bound to the target receptors. The model accounts for a saturation of receptor sites due to low specific activity  $S_{act}$  of the injected tracer. In this situation,  $k_3$  is given by  $k_{on}(B_{max} - C_2(t)/S_{act})$ , and is therefore time-dependent.  $B_{max}$  is the total receptor concentration,  $k_{on}$  the bi-molecular association rate, and  $k_4$  the dissociation rate constant  $k_{off}$ .



### Differential Equation of the Mass Balance

$$\begin{aligned}\frac{dC_1(t)}{dt} &= K_1 C_P(t) - k_2 C_1(t) - k_{on} C_1(t) \left( B_{max} - \frac{C_2(t)}{S_{act}} \right) + k_4 C_2(t) \\ \frac{dC_2(t)}{dt} &= k_{on} C_1(t) \left( B_{max} - \frac{C_2(t)}{S_{act}} \right) - k_4 C_2(t)\end{aligned}$$

with input curve  $C_P(t)$ .

### Operational Model Curve

$$C_{Model}(t) = (1 - v_B)(C_1(t) + C_2(t)) + v_B C_B(t)$$

with the concentration  $C_B(t)$  of tracer in whole blood, and the blood volume fraction  $v_B$ .

### Parameter Fitting

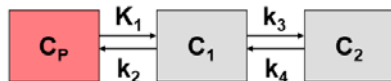
The model includes the 6 fitable parameters  $v_B$ ,  $K_1$ ,  $k_2$ ,  $k_{on}$ ,  $k_4$ ,  $B_{max}$ . Due to the high number of parameters, it is difficult to use the model for actual fitting. However, it may be valuable for generating synthetic data and assessing the effect of model simplification.

### Alternative Model with $K_1/k_2$ Fit Parameter

There is an alternative **2-Tissue Compartment Model,  $K_1/k_2$ ,  $B_{max}$**  model available which has the same structure but uses  $K_1/k_2$  as a fitting parameter instead of  $k_2$ .  $K_1/k_2$  represents the distribution volume  $V_{ND}$  of the non-displaceable compartment  $C_1$ . The advantage of this formulation is that in some situations  $V_{ND}$  can be assumed to be common among regions and estimated by a coupled fit. Alternatively,  $V_{ND}$  might already be known and can thus be fixed at a prior value.

## 2-Tissue Compartment Model, $K_1/k_2$ , $B_{max}$

The saturable receptor-ligand model includes two tissue compartments.  $C_1$  represents the non-displaceable tissue compartment with free and non-specifically bound tracer, and  $C_2$  tracer bound to the target receptors. The model accounts for a saturation of receptor sites due to low specific activity  $S_{act}$  of the injected tracer. In this situation,  $k_3$  is given by  $k_{on}(B_{max} - C_2(t)/S_{act})$ , and is therefore time-dependent.  $B_{max}$  is the total receptor concentration,  $k_{on}$  the bimolecular association rate, and  $k_4$  the dissociation rate constant  $k_{off}$ .



### Differential Equation of the Mass Balance

$$\begin{aligned}\frac{dC_1(t)}{dt} &= K_1 C_P(t) - k_2 C_1(t) - k_{on} C_1(t) \left( B_{max} - \frac{C_2(t)}{S_{act}} \right) + k_4 C_2(t) \\ \frac{dC_2(t)}{dt} &= k_{on} C_1(t) \left( B_{max} - \frac{C_2(t)}{S_{act}} \right) - k_4 C_2(t)\end{aligned}$$

with input curve  $C_p(t)$ .

### Operational Model Curve

$$C_{Model}(t) = (1 - v_B)(C_1(t) + C_2(t)) + v_B C_B(t)$$

with the concentration  $C_B(t)$  of tracer in whole blood, and the blood volume fraction  $v_B$ .

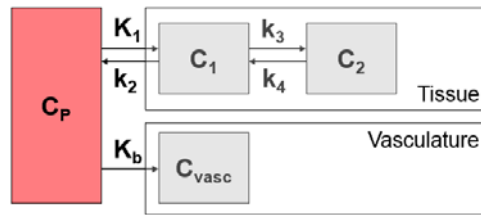
### Parameter Fitting

This model alternative **2-Tissue Compartment Model,  $K_1/k_2$ ,  $B_{max}$**  uses  $K_1/k_2$  as a fitting parameter instead of  $k_2$ .  $K_1/k_2$  represents the distribution volume  $V_{ND}$  of the non-displaceable compartment  $C_1$ . The advantage of this formulation is that in some situations  $V_{ND}$  can be assumed to be common among regions and estimated by a coupled fit. Alternatively,  $V_{ND}$  might already be known and can thus be fixed at a prior value. The other 5 fitable parameters  $v_B$ ,  $K_1$ ,  $k_3$ ,  $k_4$ ,  $B_{max}$  are the same as for the **2-Tissue Compartment Model,  $B_{max}$**  model.

Due to the high number of parameters, it is difficult to use the model for actual fitting. However, it may be valuable for generating synthetic data and assessing the effect of model simplification.

## 2-Tissue Compartment Model with Vascular Trapping

This model has been developed by Rizzo et al. [1] for the analysis of  $[^{11}C]$ -PBR28 uptake in the human brain. In addition to the compartments in the standard 2-tissue compartment model, a compartment  $C_{vasc}$  was introduced which represents slow binding of the tracer to the endothelium of the vasculature.



### Differential Equation of the Mass Balance

$$\begin{aligned} \frac{dC_1(t)}{dt} &= K_1 C_p(t) - (k_2 + k_3) C_1(t) + k_4 C_2(t) \\ \frac{dC_2(t)}{dt} &= k_3 C_1(t) - k_4 C_2(t) \\ \frac{dC_{vasc}(t)}{dt} &= K_b C_p(t) \end{aligned}$$

with input curve  $C_p(t)$ .

### Operational Model Curve

$$C_{Model}(t) = (1 - v_B)(C_1(t) + C_2(t)) + v_B(C_B(t) + C_{vasc}(t))$$

with the concentration  $C_B(t)$  of tracer in whole blood, and the blood volume fraction  $v_B$ .

### Parameter Fitting

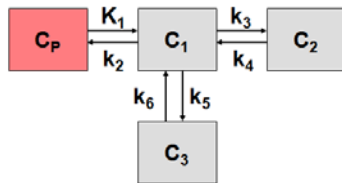
The model includes the 6 fitable parameters  $v_B$ ,  $K_1$ ,  $k_2$ ,  $k_3$ ,  $k_4$ ,  $K_b$ . Usually,  $v_B$  is fixed at a physiologic value of 3-5%. The default initial parameter values are the ones which were used for PBR28.

### Reference

1. Rizzo G, Veronese M, Tonietto M, Zanotti-Fregonara P, Turkheimer FE, Bertoldo A. Kinetic modeling without accounting for the vascular component impairs the quantification of [(11)C]PBR28 brain PET data. *J Cereb Blood Flow Metab.* 2014;34(6):1060-9. **DOI** <http://dx.doi.org/10.1038/jcbfm.2014.55>

## 3-Tissue Compartment Model

This 3-tissue compartment model separates free tracer  $C_1(t)$  and non-specifically bound tracer  $C_3(t)$  in the non-displaceable compartment.  $K_1$  and  $k_2$  describe the exchange between arterial plasma and tissue,  $k_3$  and  $k_4$  between free and specifically bound tracer  $C_2(t)$  in tissue, and  $k_5$  and  $k_6$  between free and non-specifically bound tracer.



### Differential Equation of the Mass Balance

$$\begin{aligned}\frac{dC_1(t)}{dt} &= K_1 C_p(t) - (k_2 + k_3 + k_5) C_1(t) + k_4 C_2(t) + k_6 C_3(t) \\ \frac{dC_2(t)}{dt} &= k_3 C_1(t) - k_4 C_2(t) \\ \frac{dC_3(t)}{dt} &= k_5 C_1(t) - k_6 C_3(t)\end{aligned}$$

with input curve  $C_p(t)$ .

### Operational Model Curve

$$C_{Model}(t) = (1 - v_B) (C_1(t) + C_2(t) + C_3(t)) + v_B C_B(t)$$

with the concentration  $C_B(t)$  of tracer in whole blood, and the blood volume fraction  $v_B$ .

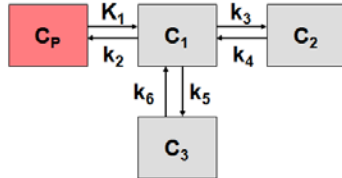
### Parameter Fitting

The model in principle includes the 7 fitable parameters  $v_B$ ,  $K_1$ ,  $k_2$ ,  $k_3$ ,  $k_4$ ,  $k_5$ ,  $k_6$ , but reliable fitting cannot be achieved in practice. However, the model can be valuable for educational purposes and for generating simulation data.



### 3-Tissue Compartment Model, $K_1/k_2$ , $V_s$

This 3-tissue compartment model separates free tracer  $C_1(t)$  and non-specifically bound tracer  $C_3(t)$  in the non-displaceable compartment.  $K_1$  and  $k_2$  describe the exchange between arterial plasma and tissue,  $k_3$  and  $k_4$  between free and specifically bound tracer  $C_2(t)$  in tissue, and  $k_5$  and  $k_6$  between free and non-specifically bound tracer.



#### Differential Equation of the Mass Balance

$$\begin{aligned}\frac{dC_1(t)}{dt} &= K_1 C_P(t) - (k_2 + k_3 + k_5) C_1(t) + k_4 C_2(t) + k_6 C_3(t) \\ \frac{dC_2(t)}{dt} &= k_3 C_1(t) - k_4 C_2(t) \\ \frac{dC_3(t)}{dt} &= k_5 C_1(t) - k_6 C_3(t)\end{aligned}$$

with input curve  $C_P(t)$ .

#### Operational Model Curve

$$C_{Model}(t) = (1 - v_B) (C_1(t) + C_2(t) + C_3(t)) + v_B C_B(t)$$

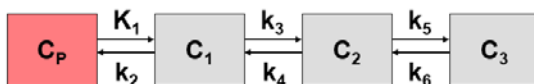
with the concentration  $C_B(t)$  of tracer in whole blood, and the blood volume fraction  $v_B$ .

#### Parameter Fitting using Distribution Volumes

This variant of the **3-Tissue Compartment Model (on page 119)** uses three distribution volumes as fitting parameters: the distribution volume of free tracer ( $K_1/k_2$ ), of non-specifically bound tracer  $V_{ns}$  ( $= (K_1 * k_5) / (k_2 * k_6)$ ), and specifically bound tracer  $V_s$  ( $= (K_1 * k_3) / (k_2 * k_4)$ ) instead of  $k_2$ ,  $k_4$  and  $k_6$ .

### 3-Tissue Compartment Model, Sequential

This model has been developed by Bertoldo et al. [1] for the analysis of FDG uptake in skeletal muscle. In this context  $C_1$ ,  $C_2$  and  $C_3$  correspond to extracellular, intracellular and phosphorylated intracellular FDG.  $K_1$  [ml/ccm/min] and  $k_2$  [1/min] are the exchange rate constants between plasma and the extracellular space,  $k_3$  [1/min] and  $k_4$  [1/min] describe the transport in and out of the cell,  $k_5$  [1/min] and  $k_6$  [1/min] phosphorylation and dephosphorylation.



### Differential Equation of the Mass Balance

$$\begin{aligned}\frac{dC_1(t)}{dt} &= K_1 C_p(t) - (k_2 + k_3) C_1(t) + k_4 C_2(t) \\ \frac{dC_2(t)}{dt} &= k_3 C_1(t) - (k_4 + k_5) C_2(t) + k_6 C_3(t) \\ \frac{dC_3(t)}{dt} &= k_5 C_2(t) - k_6 C_3(t)\end{aligned}$$

with input curve  $C_p(t)$ .

### Operational Model Curve

$$C_{Model}(t) = (1 - v_B) (C_1(t) + C_2(t) + C_3(t)) + v_B C_B(t)$$

with the concentration  $C_B(t)$  of tracer in whole blood, and the blood volume fraction  $v_B$ .

### Parameter Fitting

The model includes the 6 fitable parameters  $v_B$ ,  $K_1$ ,  $k_2$ ,  $k_3$ ,  $k_4$ ,  $k_5$ ,  $k_6$ . Usually,  $v_B$  is fixed at a physiologic value of 3-5%, and  $k_6$  is fixed:  $k_6=0$ . The influx  $K_i$  is given by

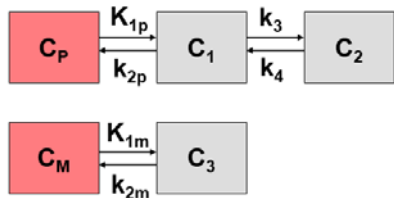
$$K_i = \frac{K_1 k_3 k_5}{k_2 k_4 + k_2 k_5 + k_3 k_5}$$

### Reference

1. Bertoldo A, Peltoniemi P, Oikonen V, Knuuti J, Nuutila P, Cobelli C: Kinetic modeling of [18F]FDG in skeletal muscle by PET: a four-compartment five-rate-constant model. American journal of physiology Endocrinology and metabolism 2001;281(3):E524-536. *PDF* <http://ajpendo.physiology.org/content/ajpendo/281/3/E524.full.pdf>

## 3-Tissue Compartments with Metabolites

If a labeled metabolite of the tracer enters tissue, the additional signal has to be accounted for in the model. The current model introduced by Fujita et al. [1] includes a second input curve  $C_M(t)$  of a metabolite entering tissue and undergoing non-specific binding.



$C_1$  represents the non-displaceable compartment of the authentic ligand,  $C_2$  the specific binding of interest, and  $C_3$  metabolized ligand in tissue.

### Differential Equation of the Mass Balance

$$\begin{aligned}\frac{dC_1(t)}{dt} &= K_{1p} C_p(t) - (k_{2p} + k_3) C_1(t) + k_4 C_2(t) \\ \frac{dC_2(t)}{dt} &= k_3 C_1(t) - k_4 C_2(t) \\ \frac{dC_3(t)}{dt} &= K_{1m} C_M(t) - k_{2m} C_3(t)\end{aligned}$$

with the input curves of authentic ligand and metabolite,  $C_p(t)$  and  $C_M(t)$ , respectively. The input curve loading menu shows two corresponding entries as soon as the model is selected.

### Operational Model Curve

$$C_{Model}(t) = (1 - v_B) (C_1(t) + C_2(t) + C_3(t)) + v_B C_B(t)$$

with the concentration  $C_B(t)$  of tracer in whole blood, and the blood volume fraction  $v_B$ .

### Parameter Fitting

The model includes the 7 fitable parameters **vB**, **K1p**, **k2p**, **k3**, **k4**, **K1m**, **k2m**. Due to the high number of parameters reliable fitting is difficult. There are two other model variants which support coupled fitting by using distribution volumes as fit parameters.

### Models with Alternative Fit Parameters

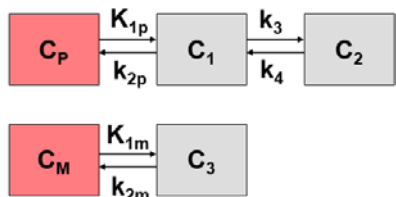
There are two alternative models available which have the same structure but use some parameter ratios as fit parameters. The advantage is that these ratios might be candidates as common parameters among regions in a coupled fit, or might be fixed at known values.

### Reference

1. Fujita M, Seibyl JP, Verhoeff NP, Ichise M, Baldwin RM, Zoghbi SS, Burger C, Staley JK, Rajeevan N, Charney DS et al: Kinetic and equilibrium analyses of [(123)I]epidepride binding to striatal and extrastriatal dopamine D(2) receptors. Synapse 1999, 34(4):290-304.

## 3-Tissue Compartments with Metabolites, K1/k2

If a labeled metabolite of the tracer enters tissue, the additional signal has to be accounted for in the model. The current model introduced by Fujita et al. [1] includes a second input curve  $C_M(t)$  of a metabolite entering tissue and undergoing non-specific binding.  $C_1$  represents the non-displaceable compartment of the authentic ligand,  $C_2$  the specific binding of interest, and  $C_3$  metabolized ligand in tissue.



## Differential Equation of the Mass Balance

$$\begin{aligned}\frac{dC_1(t)}{dt} &= K_{1p} C_p(t) - (k_{2p} + k_3) C_1(t) + k_4 C_2(t) \\ \frac{dC_2(t)}{dt} &= k_3 C_1(t) - k_4 C_2(t) \\ \frac{dC_3(t)}{dt} &= K_{1m} C_M(t) - k_{2m} C_3(t)\end{aligned}$$

with the input curves of authentic ligand and metabolite,  $C_p(t)$  and  $C_M(t)$ , respectively. The input curve loading menu shows two corresponding entries as soon as the model is selected.

## Operational Model Curve

$$C_{Model}(t) = (1 - v_B) (C_1(t) + C_2(t) + C_3(t)) + v_B C_B(t)$$

with the concentration  $C_B(t)$  of tracer in whole blood, and the blood volume fraction  $v_B$ .

## Parameter Fitting using $K_1/k_2$

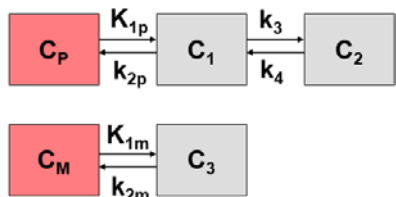
The **3-Tissue Compartments with Metabolites, K1/k2** model variant uses the parameter ratios **K1p/k2p**, **K1m/k2m** and **K1p/K1m**, as fitting parameters instead of  $k_{2p}$ ,  $K_{1m}$  and  $k_{2m}$ . The advantage of this formulation is that the ratios may be fixed at known or assumed values, or estimated in a coupled fit. The other four fitting parameters **vB**, **K1p**, **k3**, and **k4** are the same as for the **3-Tissue Compartments with Metabolites** (on page 121) model.

## Reference

1. Fujita M, Seibyl JP, Verhoeff NP, Ichise M, Baldwin RM, Zoghbi SS, Burger C, Staley JK, Rajeevan N, Charney DS et al: Kinetic and equilibrium analyses of [(123)I]epidepride binding to striatal and extrastriatal dopamine D(2) receptors. Synapse 1999, 34(4):290-304.

## 3-Tissue Compartments with Metabolites, K1/k2, Vs

If a labeled metabolite of the tracer enters tissue, the additional signal has to be accounted for in the model. The current model introduced by Fujita et al. [1] includes a second input curve  $C_M(t)$  of a metabolite entering tissue and undergoing non-specific binding.  $C_1$  represents the non-displaceable compartment of the authentic ligand,  $C_2$  the specific binding of interest, and  $C_3$  metabolized ligand in tissue.



## Differential Equation of the Mass Balance

$$\begin{aligned}\frac{dC_1(t)}{dt} &= K_{1p} C_p(t) - (k_{2p} + k_3) C_1(t) + k_4 C_2(t) \\ \frac{dC_2(t)}{dt} &= k_3 C_1(t) - k_4 C_2(t) \\ \frac{dC_3(t)}{dt} &= K_{1m} C_M(t) - k_{2m} C_3(t)\end{aligned}$$

with the input curves of authentic ligand and metabolite,  $C_p(t)$  and  $C_M(t)$ , respectively. The input curve loading menu shows two corresponding entries as soon as the model is selected.

## Operational Model Curve

$$C_{Model}(t) = (1 - v_B) (C_1(t) + C_2(t) + C_3(t)) + v_B C_B(t)$$

with the concentration  $C_B(t)$  of tracer in whole blood, and the blood volume fraction  $v_B$ .

## Parameter Fitting using $K_1/k_2$ and VS

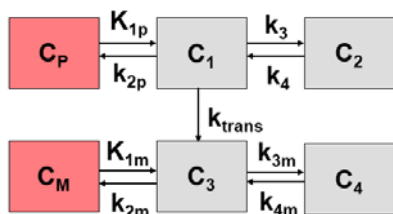
The **3-Tissue Compartments with Metabolites, K1/k2, Vs** model variant uses the parameter ratios  $K_{1p}/k_{2p}$ ,  $K_{1m}/k_{2m}$ ,  $K_{1p}/K_{1m}$ , and  $Vs = K_{1p}/k_{2p} * k_3/k_4$  as fitting parameters instead of  $k_{2p}$ ,  $K_{1m}$ ,  $k_{2m}$ , and  $k_4$ . The advantage of this formulation is that the ratios may be fixed at known or assumed values, or estimated in a coupled fit. The other three fitting parameters  $v_B$ ,  $K_{1p}$  and  $k_3$  are the same as for the **3-Tissue Compartments with Metabolites** (on page 121) model.

## Reference

1. Fujita M, Seibyl JP, Verhoeff NP, Ichise M, Baldwin RM, Zoghbi SS, Burger C, Staley JK, Rajeevan N, Charney DS et al: Kinetic and equilibrium analyses of [(123)I]epidepride binding to striatal and extrastriatal dopamine D(2) receptors. Synapse 1999, 34(4):290-304.

## 4-Tissue Compartments with Metabolites, K1/k2

This is a general compartment model for ligands which have recirculating metabolites entering tissue and contributing to tissue activity. Tissue kinetics of both the authentic ligand ( $C_1$  and  $C_2$ ) as well as the metabolites ( $C_3$  and  $C_4$ ) are described by a 2-tissue compartment model, with a transfer  $k_{trans}$  constant linking the two exchangeable compartments which represents metabolites generated in tissue.



### Differential Equation of the Mass Balance

$$\begin{aligned}\frac{dC_1(t)}{dt} &= K_{1p}C_p(t) - (k_{2p} + k_3 + k_{trans})C_1(t) + k_4C_2(t) \\ \frac{dC_2(t)}{dt} &= k_3C_1(t) - k_4C_2(t) \\ \frac{dC_3(t)}{dt} &= K_{lm}C_M(t) - (k_{2m} + k_{3m})C_3(t) + k_{trans}C_1(t) + k_{4m}C_4(t) \\ \frac{dC_4(t)}{dt} &= k_{3m}C_3(t) - k_{4m}C_4(t)\end{aligned}$$

Two input curves are required: the authentic ligand in plasma  $C_p(t)$ , and the metabolites exchanging with tissue  $C_M(t)$ . The input curve loading menu shows two corresponding entries as soon as the model is selected.

### Operational Model Curve

$$C_{Model}(t) = (1 - v_B)(C_1(t) + C_2(t) + C_3(t) + C_4(t)) + v_B C_B(t)$$

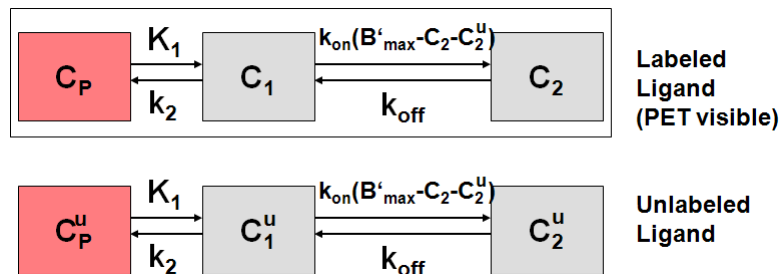
with the concentration  $C_B(t)$  of tracer in whole blood, and the blood volume fraction  $v_B$ .

### Parameter Fitting

The model includes more parameters than can be reliably fitted at once. Therefore it has been implemented with **K1/k2** and **K1m/k2m** as fitting parameters to allow fixing the distribution volume of the exchangeable compartment, or to use them as common parameters in a coupled fit.

## Flumazenil Triple-injection Model

This model corresponds to a triple-injection protocol developed by Delforge et al. [1] for [11C]flumazenil PET. The setup includes three injections of labeled and/or unlabeled ligand during a single acquisition. The model is based on a 2-tissue compartment structure and accounts for the saturation of the benzodiazepine receptor. The uptake and binding of both labeled and unlabeled ligand is modeled by two parallel compartment models linked by common model parameters.



## Differential Equation of the Mass Balance

They are linked by common model parameters and the system of differential equations

$$\begin{aligned}\frac{dC_1(t)}{dt} &= k_1 C_p(t) - k_2 C_1(t) - \frac{k_{on}}{V_R} C_1(t) [B_{max} - C_2(t) - C_2^u(t)] + k_{off} C_2(t) \\ \frac{dC_2(t)}{dt} &= \frac{k_{on}}{V_R} C_1(t) [B_{max} - C_2(t) - C_2^u(t)] - k_{off} C_2(t) \\ \frac{dC_1^u(t)}{dt} &= k_1 C_p^u(t) - k_2 C_1^u(t) - \frac{k_{on}}{V_R} C_1^u(t) [B_{max} - C_2(t) - C_2^u(t)] + k_{off} C_2^u(t) \\ \frac{dC_2^u(t)}{dt} &= \frac{k_{on}}{V_R} C_1^u(t) [B_{max} - C_2(t) - C_2^u(t)] - k_{off} C_2^u(t)\end{aligned}$$

Note that only the labeled ligand can be measured, both in PET and in the blood samples. Therefore the input curve of unlabeled ligand  $C_p^u(t)$  must be derived from the input curves of labeled ligand  $C_p(t)$ . This task is complicated by the fact of metabolite build-up in plasma which must be corrected for. With  $T_1$ ,  $T_2$  and  $T_3$  denoting the time of the three injections, the input curve  $C_p(t)$  of hot (labeled) tracer is calculated from the total plasma activity  $C_{plasma}(t)$  as follows:

$$C_p(t) = \begin{cases} C_{plasma} (A_1 e^{B_1(t-T_1)} + A_2 e^{B_2(t-T_1)}) & T_1 < t < T_3 \quad \text{decay since 1st injection} \\ C_{plasma} (A_1 e^{B_1(t-T_3)} + A_2 e^{B_2(t-T_3)}) & t > T_3 \quad \text{decay since 3rd injection} \end{cases}$$

This metabolite correction is implemented as the input curve model **Multiinjection, HOT**.

The input curve  $C_p^u(t)$  of cold (unlabeled) tracer is derived from  $C_p(t)$  by the expression

$$C_p^u(t) = \begin{cases} Dose_2 / Dose_1 C_p(t - T_2) & T_2 < t < T_3 \\ Dose_3^{Cold} / Dose_3^{Hot} C_p(t - T_3) + C_p^u(T_3) (A_1 e^{B_1(t-T_3)} + A_2 e^{B_2(t-T_3)}) & t > T_3 \end{cases}$$

which is implemented as the input curve model **Multiinjection, COLD**.

## Operational Model Curve

Since PET only measures the radioactively labeled compounds the operational equation is given by

$$C_{Model}(t) = (1 - v_B) (C_1(t) + C_2(t)) + v_B C_B(t)$$

with the concentration  $C_B(t)$  of labeled compounds in whole blood, and the blood volume fraction  $v_B$ .

## Parameter Fitting

The model includes six parameters **vB**, **K1**, **k2**, **Kon/VR**, **koff**, **Bmax** which are normally not possible to fit at once. However, the complex injection protocol results in more complex tissue TACs than with the usual single-injection protocols. As a consequence, the model has proven to result in reliable estimates of  $B_{max}$ .

## Reference

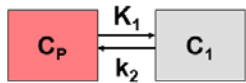
1. Delforge J, Pappata S, Millet P, Samson Y, Bendriem B, Jobert A, Crouzel C, Syrota A: Quantification of benzodiazepine receptors in human brain using PET, [11C]flumazenil, and a single-experiment protocol. J Cereb Blood Flow Metab 1995, 15(2):284-300. DOI <http://www.nature.com/jcbfm/journal/v15/n2/pdf/jcbfm199534a.pdf>

## Acknowledgement

The model implementation was supported in helpful discussions by Dr. Philippe Millet, University Hospital Geneva, Switzerland.

## Water PET Model with Flow and Dispersion

This model implements the 1-tissue compartment model for H<sub>2</sub><sup>15</sup>O water PET studies developed by E. Meyer [1].



## Operational Model Curve

It is assumed that at by the arrival in tissue the true input curve  $C_p(t)$  has been dispersed. This dispersion is modeled by convolving  $C_p(t)$  with an exponential kernel

$$d(t) = \frac{1}{\tau} e^{-t/\tau}$$

whereby the parameter  $\tau$  is called *Dispersion*. Incorporating the convolution into the 1-tissue compartment model and solving it using the Laplace transform yields the following analytic formula for the tissue concentration

$$C_1(t) = \tau K_1 C_p(t) + (1 - \tau k_2) K_1 e^{-k_2 t} \int_0^t C_p(\tau) e^{k_2 \tau} d\tau$$

$K_1$  represents flow, and  $k_2$  flow divided by the partition coefficient  $p$ .

The operational equation is given by

$$C_{Model}(t) = (1 - vB) C_1(t) + vB C_B(t)$$

Note that in this particular case there is no difference between the plasma and blood concentrations  $C_p(t)$  and  $C_B(t)$  because water is a freely diffusible tracer.

## Parameter Fitting

The tissue model includes four fitable parameters  $vB$ ,  $K1$ ,  $k2$ , (Flow/Part.Coeff),  $\tau$  (Dispersion). Per default, however,  $v_B$  is fixed at  $vB=0$  as in the original model. For water studies with the blood activity sampled in a peripheral artery, the resulting input curve is



delayed respect to the arrival in tissue. In this case, the model should be fitted using the **Fit region and blood delay** (on page 77) button.

### Implementation Note

Unlike the standard compartment models in PKIN the model curve of the operational equation is not calculated by integrating a set of differential equations, but rather using the analytical solution given above. The convolution integral is approximated by summing rectangles of 0.1sec in length, assuming that the acquisition time of dynamic water studies is short.

### Reference

1. Meyer E: Simultaneous correction for tracer arrival delay and dispersion in CBF measurements by the H215O autoradiographic method and dynamic PET. J Nucl Med 1989, 30(6):1069-1078. *PDF* <http://jnm.snmjournals.org/content/30/6/1069.full.pdf>

## Compartment Models for Cardiac Perfusion PET

Model Name	Description
<i>Card. NH3 (de Grado)</i> (on page 130)	1-Tissue compartment model with two spillover terms. Has been developed for cardiac $\text{NH}_3$ -PET studies with bolus administration. The left ventricle input curve is linearly corrected for metabolites.
<i>Card. NH3 (UCLA, 2 Parameters)</i> (on page 131)	2-Tissue compartment model with only 2 fitted parameters. Has been developed for cardiac $\text{NH}_3$ -PET studies with bolus administration. It only uses the first 2 minutes of the data as there is not metabolite correction included.
<i>Card. NH3 (2 Compartments)</i> (on page 133)	2-Tissue compartment model with metabolic trapping. Has been developed for cardiac $\text{NH}_3$ -PET studies with bolus administration. Also implements metabolite correction.
<i>Card. Rb-82 (1 Compartment)</i> (on page 136)	1-compartment model for the quantification of myocardial perfusion with Rubidium-82 PET data.
<i>Card. Rb-82 (2 Compartment)</i> (on page 137)	Two-compartment model for the quantification of myocardial perfusion with Rubidium-82 PET data.
<i>Card. H2O (Tissue fraction)</i> (on page 139)	1-Tissue compartment model with two spillover terms. Has been developed for cardiac $\text{H}_2^{15}\text{O}$ - PET studies with bolus administration.
Card. H2O (Geometrical corr.)	As above, but with geometrical spillover correction.
<i>Card. Acetate (1 Compartment)</i> (on page 140)	1-compartment model for the quantification of myocardial perfusion with $^{11}\text{C}$ Acetate PET data.

### Assumptions of the Cardiac Models

- ▶ The cardiac models usually apply a geometrical spillover with two correction terms: correction for signal from the left and right ventricular cavity.
- ▶ For calculating the myocardial perfusion per g tissue a density of 1.04 g/ml for myocardial tissue is applied.
- ▶ Usually the PET signal from the left ventricle or the atrium is used as a measure of the tracer activity in blood. Depending on the tracer, corrections are applied to this blood curve for calculation the concentration of parent tracer with is required as the input curve of the models.

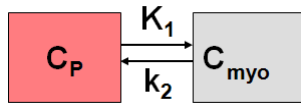
Exceptions to these rules are specified in the description of the individual models.

## Cardiac Flow from Ammonia PET

There are three model variants available for the quantification of myocardial blood flow from  $^{13}\text{NH}_3$  ammonia bolus PET data, the 1-tissue compartment model and the 2-tissue compartment model with metabolic trapping. The 1-tissue model is preferably used with the first 2-4 minutes of a dynamic measurement, while the 2-tissue model is also adequate for longer durations. This may compensate to some extent the increased vulnerability to identifiability problems due to the higher number of fit parameters in the 2-tissue model.

### Card. NH3 (de Grado)

The **Card. NH3(De Grado)** model has been developed by DeGrado et al. [1] for cardiac PET studies using  $^{13}\text{NH}_3$  ammonia bolus injection. It is based on a 1-tissue compartment model with plasma activity  $C_p(t)$  and total uptake in myocardium  $C_{\text{myo}}(t)$ .



### Operational Model Curve

The plasma activity is calculated from the total activity in the left ventricular cavity  $C_{lv}(t)$  by a linear metabolite correction with factor mCorr

$$C_p(t) = (1 - mCorr * t) C_{lv}(t)$$

A value of  $mCorr=0.077$  [1/min] has been found in humans [1]. The resulting differential equation for myocardial uptake is

$$\frac{dC_{\text{myo}}(t)}{dt} = K_1 C_p(t) - k_2 C_{\text{myo}}(t)$$

where  $K_1$  equals myocardial blood flow  $F$ . The model curve incorporates a cardiac dual spillover correction, resulting in the operational equation

$$C_{\text{Model}}(t) = (1 - V_{LV} - V_{RV}) C_{\text{myo}}(t) + V_{RV} C_{LV}(t) + V_{RV} C_{RV}(t)$$

where

$V_{LV}$  = spill-over fraction of the blood activity in the left ventricle  $C_{LV}(t)$ ,

$V_{RV}$  = spill-over fraction of the blood activity in the right ventricle  $C_{RV}(t)$ .

DeGrado et al. recommend to only use the first 4 minutes of data after injection of the tracer to reduce the effects of metabolite buildup and washout.

### Implementation

When using the model from the PCARD module, the data are transferred appropriately. When using it in PKIN the blood data have to be loaded as follows:

- ▶ The left ventricle curve must be loaded as the **Plasma activity** curve. No metabolite correction needs to be enabled on the **Blood** panel of PKIN because it is included in the tissue model.
- ▶ The right ventricle curve must be loaded as the **Whole blood** curve.

The following automatic adjustments are performed within the model:

- ▶ The spill-over fraction from the right ventricle  $V_{RV}$  is automatically fixed to zero if the string "Sep" is *not* contained in the name a region. The assumption is that such a TAC is not from septal tissue and should thus be modeled with spill-over from the left ventricle only.
- ▶ The **valid** flag for all data samples after 4 minutes is set to false, and they are consequently not considered in the fit.

## Parameter Fitting

The model includes the 4 fitable parameters **F**, **vRV**, **vLV** and **k2**. Please inspect the %SE standard error to get information about the reliability of the parameter estimates.

## Reference

1. DeGrado TR, Hanson MW, Turkington TG, DeLong DM, Brezinski DA, Vallee JP, Hedlund LW, Zhang J, Cobb F, Sullivan MJ et al: Estimation of myocardial blood flow for longitudinal studies with  $^{13}\text{N}$ -labeled ammonia and positron emission tomography. J Nucl Cardiol 1996, 3(6 Pt 1):494-507.

## Card. NH3 (UCLA, 2 Parameters)

This model implements a 2-tissue compartment model with only two fitted tissue rate constants. It was developed by Choi et al. for cardiac PET studies using  $^{13}\text{NH}_3$  ammonia bolus injection and was called the *Modified Two-Parameter Model*, [1] eq (5).



## Operational Model Curve

The system of compartmental differential equations for this compartment structure is given by

$$\begin{aligned}\frac{dC_1(t)}{dt} &= K_1 C_p(t) - (k_2 + k_3) C_1(t) \\ \frac{dC_2(t)}{dt} &= k_3 C_1(t)\end{aligned}$$

with  $K_1$  corresponding to myocardial blood flow  $F$ . The  $k_3$  rate constant corresponding to glutamine formation was converted into a  $K_1$ -dependent value by the relation

$$k_3 = K_1 (1.65 e^{1.25/K_1} - 1)$$

Similarly, the  $k_2$  rate constant of ammonia washout was converted into a  $K_1$ -dependent value by

$$k_2 = K_1 / V_{ND}$$

with the  $V_{ND}$  representing the distribution volume of ammonia in the  $C_1$  compartment which was assumed to be constant ( $V_{ND}=0.8$ ).

In contrast to Choi's method, the model incorporates not only a spillover from the left ventricle, but for septal regions also from the right ventricle. The operational equation is then given by

$$C_{Model}(t) = (1 - V_{LV} - V_{RV})(C_1(t) + C_2(t)) + V_{LV}C_{LV}(t) + V_{RV}C_{RV}(t)$$

where

$V_{LV}$  = spill-over fraction of the blood activity in the left ventricle  $C_{LV}(t)$ ,

$V_{RV}$  = spill-over fraction of the blood activity in the right ventricle  $C_{RV}(t)$ .

Because the model does not include a metabolite correction, only the samples within 2 minutes after injection should be used for the fit [1].

### Implementation Notes:

When using the model from the PCARD module, the data are transferred appropriately. When using it in PKIN the blood data have to be loaded as follows:

- ▶▶ The left ventricle curve must be loaded as the **Plasma activity** curve.
- ▶▶ The right ventricle curve must be loaded as the **Whole blood** curve.

The following automatic adjustments are performed within the model:

- ▶▶ The spill-over fraction from the right ventricle  $V_{RV}$  is automatically fixed to zero if the string "Sep" is *not* contained in the name a region. The assumption is that such a TAC is not from septal tissue and should thus be modeled with spill-over from the left ventricle only.
- ▶▶ The **valid** flag for all data samples after 2 minutes is set to false, and they are consequently not considered in the fit.

### Parameter Fitting

The model includes the three fitable parameters **F**, **vLV**, **vRV**. Please inspect the %SE standard error to get information about the reliability of their estimates. The input parameters **Vnd** (0.8), **EF scale** (1.65) and **EF Exp** (1.25) are related to the conversion of  $k_2$  and  $k_3$  as described above and can be adjusted if needed.

### Reference

1. Choi Y, Huang SC, Hawkins RA, Kim JY, Kim BT, Hoh CK, Chen K, Phelps ME, Schelbert HR: Quantification of myocardial blood flow using  $^{13}\text{N}$ -ammonia and PET: comparison of tracer models. J Nucl Med 1999, 40(6):1045-1055. *PDF*  
<http://jnm.snmjournals.org/content/40/6/1045.full.pdf>

## Card. NH3 (2 Compartments)

The **Card NH3 (2-Tissue)** model developed by Hutchins et al. [1] is an implementation of the irreversible 2-tissue-compartment model for cardiac PET studies using  $^{13}\text{NH}_3$  ammonia bolus injection. The compartment model has the following structure



where  $C_1$  is free tracer in tissue, and  $C_2$  is metabolically trapped tracer in the form of  $^{13}\text{N}$  glutamine. Because ammonia is considered in this model as freely diffusible across the capillary wall, the unidirectional uptake parameter  $K_1$  equals the myocardial perfusion.

## Operational Model Curve

The system of differential equations is

$$\begin{aligned} \frac{dC_1(t)}{dt} &= K_1 C_P(t) - (k_2 + k_3) C_1(t) \\ \frac{dC_2(t)}{dt} &= k_3 C_1(t) \end{aligned}$$

To allow the fitting of data over an extended period, the model includes the exponential metabolite correction described by van den Hoff et al. [2]

$$C_P(t) = \begin{cases} C_{LV}(t) & t \leq t_0 \\ e^{-\ln 2(t-t_0)/T_{1/2}} C_{LV}(t) & t > t_0 \end{cases}$$

with a delay  $t_0=0.48$  min and half-time  $T_{1/2}=6.69$  min.  $C_{LV}(t)$  is the total tracer concentration measured in the left ventricular cavity, including metabolites.

Additionally, the model incorporates a cardiac dual spillover correction by the operational equation

$$C_{Model}(t) = (1 - V_{LV} - V_{RV})(C_1(t) + C_2(t)) + V_{LV}C_{LV}(t) + V_{RV}C_{RV}(t)$$

where

$V_{LV}$  = spill-over fraction of the blood activity in the left ventricle  $C_{LV}(t)$ ,

$V_{RV}$  = spill-over fraction of the blood activity in the right ventricle  $C_{RV}(t)$ .

## Implementation Notes:

When using the model from the PCARD module, the data are transferred appropriately. When using it in PKIN the blood data have to be loaded as follows:

- ▶ The left ventricle curve must be loaded as the **Plasma activity** curve. No metabolite correction needs to be enabled on the **Blood** panel of PKIN because it is included in the tissue model.
- ▶ The right ventricle curve must be loaded as the **Whole blood** curve.

The following automatic adjustments are performed within the model:

- ▶ The spill-over fraction from the right ventricle  $V_{RV}$  is automatically fixed to zero if the string "Sep" is *not* contained in the name a region. The assumption is that such a TAC is not from septal tissue and should thus be modeled with spill-over from the left ventricle only.
- ▶ The **valid** flag for all data samples after 2 minutes is set to false, and they are consequently not considered in the fit.

## Parameter Fitting

The model includes the five fitable parameters **F**, **vLV**, **vRV**, **k2**, **k3**. Please inspect the %SE standard error to get information about the reliability of their estimates. The input parameters **MC T0** (0.48) and **MC T12** (6.69) are related to the metabolite correction as described above and can be adjusted if needed.

## Alternate Model Card NH3 (2-Tissue, $K_1/k_2$ )

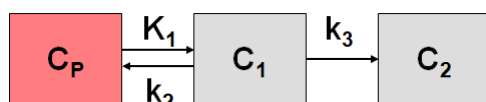
Due to the increased number of fit parameters it has been found, that the **Card NH3 (2-Tissue)** may suffer from identifiability problems. Therefore, the variant **Card NH3 (2-Tissue,  $K_1/k_2$ )** (on page 134) has been developed using the parameter  $K_1/k_2$  (distribution volume of the first compartment) as a fit parameter instead of  $k_2$ . In this configuration physiological restrictions can be imposed on  $K_1/k_2$ , or  $K_1/k_2$  can be used as a common parameter in a *coupled fit* (on page 51).

## References

1. Hutchins GD, Schwaiger M, Rosenspire KC, Krivokapich J, Schelbert H, Kuhl DE: Noninvasive quantification of regional blood flow in the human heart using N-13 ammonia and dynamic positron emission tomographic imaging. J Am Coll Cardiol 1990, 15(5):1032-1042.
2. van den Hoff J, Burchert W, Borner AR, Fricke H, Kuhnel G, Meyer GJ, Otto D, Weckesser E, Wolpers HG, Knapp WH: [1-(11)C]Acetate as a quantitative perfusion tracer in myocardial PET. J Nucl Med 2001, 42(8):1174-1182. *PDF*  
<http://jnm.snmjournals.org/content/42/8/1174.full.pdf>

## Card. NH3 (2 Compartments, $K_1/k_2$ )

The **Card NH3 (2-Tissue)** model developed by Hutchins et al. [1] is an implementation of the irreversible 2-tissue-compartment model for cardiac PET studies using  $^{13}\text{NH}_3$  ammonia bolus injection. The compartment model has the following structure



where  $C_1$  is free tracer in tissue, and  $C_2$  is metabolically trapped tracer in the form of  $^{13}\text{N}$  glutamine. Because ammonia is considered in this model as freely diffusible across the capillary wall, the unidirectional uptake parameter  $K_1$  equals the myocardial perfusion.

## Operational Model Curve

The system of differential equations is

$$\begin{aligned}\frac{dC_1(t)}{dt} &= K_1 C_P(t) - (k_2 + k_3) C_1(t) \\ \frac{dC_2(t)}{dt} &= k_3 C_1(t)\end{aligned}$$

To allow the fitting of data over an extended period, the model includes the exponential metabolite correction described by van den Hoff et al. [2]

$$C_P(t) = \begin{cases} C_{LV}(t) & t \leq t_0 \\ e^{-\ln 2(t-t_0)/T_{1/2}} C_{LV}(t) & t > t_0 \end{cases}$$

with a delay  $t_0=0.48$  min and half-time  $T_{1/2}=6.69$  min.  $C_{LV}(t)$  is the total tracer concentration measured in the left ventricular cavity, including metabolites.

Additionally, the model incorporates a cardiac dual spillover correction by the operational equation

$$C_{Model}(t) = (1 - V_{LV} - V_{RV})(C_1(t) + C_2(t)) + V_{LV} C_{LV}(t) + V_{RV} C_{RV}(t)$$

where

$V_{LV}$  = spill-over fraction of the blood activity in the left ventricle  $C_{LV}(t)$ ,

$V_{RV}$  = spill-over fraction of the blood activity in the right ventricle  $C_{RV}(t)$ .

## Implementation Notes:

When using the model from the PCARD module, the data are transferred appropriately. When using it in PKIN the blood data have to be loaded as follows:

- ▶ The left ventricle curve must be loaded as the **Plasma activity** curve. No metabolite correction needs to be enabled on the **Blood** panel of PKIN because it is included in the tissue model.
- ▶ The right ventricle curve must be loaded as the **Whole blood** curve.

The following automatic adjustments are performed within the model:

- ▶ The spill-over fraction from the right ventricle  $V_{RV}$  is automatically fixed to zero if the string "Sep" is *not* contained in the name a region. The assumption is that such a TAC is not from septal tissue and should thus be modeled with spill-over from the left ventricle only.
- ▶ The **valid** flag for all data samples after 2 minutes is set to false, and they are consequently not considered in the fit.

## Parameter Fitting

The model includes the five fitable parameters **F**, **vLV**, **vRV**, **K1/k2**, **k3**. The parameter **K1/k2** (distribution volume of the first compartment) is used as a fit parameter instead of  $k_2$ . In this configuration physiological restrictions can be imposed on  $K_1/k_2$ , or  $K_1/k_2$  can be used as a common parameter in a *coupled fit* (on page 51). **k2** is derived from the estimated **F** ( $=K_1$ )



and **K1/k2**. Please inspect the %SE standard error to get information about the reliability of their estimates.

The input parameters **MC T0** (0.48) and **MC T12** (6.69) are related to the metabolite correction as described above and can be adjusted if needed.

## References

1. Hutchins GD, Schwaiger M, Rosenspire KC, Krivokapich J, Schelbert H, Kuhl DE: Noninvasive quantification of regional blood flow in the human heart using N-13 ammonia and dynamic positron emission tomographic imaging. J Am Coll Cardiol 1990, 15(5):1032-1042.
2. van den Hoff J, Burchert W, Borner AR, Fricke H, Kuhnel G, Meyer GJ, Otto D, Weckesser E, Wolpers HG, Knapp WH: [1-(11)C]Acetate as a quantitative perfusion tracer in myocardial PET. J Nucl Med 2001, 42(8):1174-1182. *PDF*  
<http://jnm.snmjournals.org/content/42/8/1174.full.pdf>

## Cardiac Flow from Rubidium-82 PET

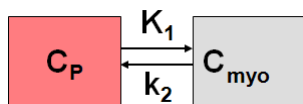
Two kinetic models are available for the quantification of myocardial perfusion from <sup>82</sup>Rb bolus PET data.

- 1) A 1-tissue compartment model with geometrical spillover correction and correction for flow-dependent extraction.

A 2-tissue compartment model with recovery and spillover correction.

### Card. Rb-82 (1 Compartment)

The **Card. Rb82(1 Compartment)** model has been developed by Lortie et al. [1] for cardiac PET studies using <sup>82</sup>Rb bolus injection. It is based on a 1-tissue compartment model with plasma activity  $C_p(t)$  and total uptake in myocardium  $C_{myo}(t)$ .



### Operational Model Curve

The differential equation is given by

$$\frac{dC_{myo}(t)}{dt} = K_1 C_P(t) - k_2 C_{myo}(t)$$

The model uses the activity in the left ventricular cavity  $C_{lv}(t)$  is used as the input curve ( $C_P(t) = C_{lv}(t)$ ). Rb is known to have a flow-dependent extraction fraction, so that  $K_1$ , which is the product of flow  $F$  times extraction fraction  $E$ , is described by the expression

$$K_1 = F E = F (1 - A e^{-B/F})$$

This expression is inserted for  $K_1$  into the differential equation, so that  $F$  becomes a fit parameter, and  $K_1$  is derived from it. The values of the extraction parameters reported by Lortie et al. [1] are

$$\begin{aligned} A &= 0.77 \\ B &= 0.63 \text{ [ml/min/g]} \end{aligned}$$

They can be entered as input parameters into the model for changing the form of the extraction function. The model curve incorporates a cardiac dual spillover correction, resulting in the operational equation

$$C_{Model}(t) = (1 - V_{LV} - V_{RV}) C_{myo}(t) + V_{RV} C_{LV}(t) + V_{RV} C_{RV}(t)$$

where

$$\begin{aligned} V_{LV} &= \text{spill-over fraction of the blood activity in the left ventricular cavity } C_{LV}(t), \\ V_{RV} &= \text{spill-over fraction of the blood activity in the right ventricular cavity } C_{RV}(t) \end{aligned}$$

## Implementation

When using the model from the PCARD module, the data are transferred appropriately. When using it in PKIN the blood data have to be loaded as follows:

- ▶▶ The left ventricle curve must be loaded as the **Plasma activity** curve.
- ▶▶ The right ventricle curve must be loaded as the **Whole blood** curve.

The following automatic adjustment is performed within the model:

- ▶▶ The spill-over fraction from the right ventricle  $V_{RV}$  is automatically fixed to zero if the string "Sep" is *not* contained in the name a region. The assumption is that such a TAC is not from septal tissue and should thus be modeled with spill-over from the left ventricle only.

## Parameter Fitting

The model includes the 4 fitable parameters **F**, **vRV**, **vLV** and **k2**. Please inspect the %SE standard error to get information about the reliability of the parameter estimates. The scale factor **A (EF scale)** and the exponent **B (EF exp)** of the extraction function can be entered as input parameters.

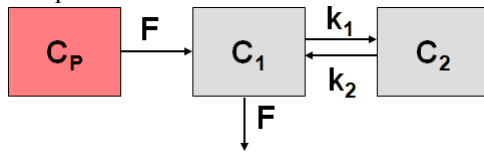
## Reference

1. Lortie M, Beanlands RS, Yoshinaga K, Klein R, Dasilva JN, DeKemp RA: Quantification of myocardial blood flow with 82Rb dynamic PET imaging. Eur J Nucl Med Mol Imaging 2007, 34(11):1765-1774.

## Card. Rb-82 (2 Compartments)

The **Card. Rb-82 (2 Compartments)** model has been implemented according to the method described and evaluated by Herrero et al [1]. The model is based on the following

compartment structure to describe the kinetics of rubidium in the myocardium:



where  $C_1(t)$  represents the fast exchangeable compartment (vascular and interstitial spaces), and  $C_2(t)$  the slow exchangeable compartment (intracellular space), myocardium flow  $F$ , and rate constants  $k_1$  and  $k_2$ .

### Operational Model Curve

The differential equations for the activity concentrations in the different compartments are given by

$$\begin{aligned}\frac{dC_1(t)}{dt} &= F(C_P(t) - C_1(t)) - k_1 C_1(t) + k_2 C_2(t) \\ \frac{dC_2(t)}{dt} &= k_1 C_1(t) - k_2 C_2(t)\end{aligned}$$

The model uses the activity in the left ventricular cavity  $C_{LV}(t)$  is used as the input curve ( $C_P(t) = C_{LV}(t)$ ). The operational equation which is fitted to the measured data is

$$C_{Model} = F_{MM} (C_1(t) + C_2(t)) + F_{BM} C_{LV}(t)$$

where  $F_{MM}$  denotes the tissue recovery coefficient and  $F_{BM}$  the blood to myocardium spillover fraction.

### Implementation

When using the model from the PCARD module, the data are transferred appropriately. When using it in PKIN the left ventricle curve must be loaded as the **Plasma activity** curve. The right ventricle curve is not used.

### Parameter Fitting

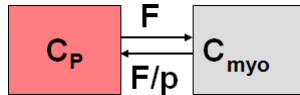
The model encompasses 5 fitable parameters. However, in practice it is impossible to estimate so many parameters from a time-activity curve with reasonable identifiability. Therefore, at least the recovery coefficient **FMM** is usually fixed. The default from Herrero et al [1] is 0.65, but it depends on the image resolution and should be determined experimentally.

### Reference

1. Herrero P, Markham J, Shelton ME, Bergmann SR: Implementation and evaluation of a two-compartment model for quantification of myocardial perfusion with rubidium-82 and positron emission tomography. *Circulation research* 1992, 70(3):496-507. DOI <http://dx.doi.org/10.1161/01.RES.70.3.496>

## Cardiac Flow from Water PET

The **Card. H<sub>2</sub>O (Tissue fraction)** model has been developed by Hermannsen et al. [1] for cardiac PET studies using H<sub>2</sub><sup>15</sup>O water bolus injection. The distribution of the freely diffusible inert tracer H<sub>2</sub><sup>15</sup>O in myocardium can be described by a 1-tissue compartment model with myocardial blood flow  $F$  and the partition coefficient of water  $p$ , which is defined as (ml water/ml tissue)/(ml water/ml blood).



### Operational Model Curve

The differential equation for the tracer concentration in myocardium is

$$\frac{dC_{myo}(t)}{dt} = FC_p(t) - \frac{F}{p}C_{myo}(t)$$

In practice, the left ventricular time-activity curve is used as the input curve  $C_p(t)$ .

The model curve incorporates two spill-over terms from blood in the left and the right ventricular cavities which are relatively displaced in time. So the operational equation which is fitted to the measured data is

$$C_{Model}(t) = TF C_{myo}(t) + V_{LV} C_{LV}(t) + V_{RV} C_{RV}(t)$$

where  $TF$  = tissue fraction,  $V_{LV}$  = spill-over fraction from the left ventricle,  $V_{RV}$  = spill-over fraction from the right ventricle.

### Implementation

As opposed to most models in PKIN a tissue fraction is used for calculating the model curve rather than a strict geometrical correction for compliance with the original model. When using the model from the PCARD module, the data are transferred appropriately. When using it in PKIN the blood data have to be loaded as follows:

- ▶▶ The left ventricle curve must be loaded as the **Plasma activity** curve.
- ▶▶ The right ventricle curve must be loaded as the **Whole blood** curve.

The following automatic adjustment is performed within the model:

- ▶▶ The spill-over fraction from the right ventricle  $V_{RV}$  is automatically fixed to zero if the string "Sep" is *not* contained in the name of a region. The assumption is that such a TAC is not from septal tissue and should thus be modeled with spill-over from the left ventricle only.

### Parameter Fitting

The model encompasses the 5 fitable parameters  $F$ ,  $TF$ ,  $vLV$ ,  $vRV$  and  $p$ . However, in practice it is impossible to estimate so many parameters from a time-activity curve with reasonable identifiability. The partition coefficient is therefore fixed per default at a value of

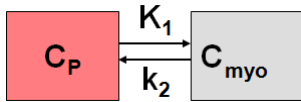
0.96 [1], and the spillover fraction from right ventricular blood vRV is only fitted for septal tissue TACs.

## Reference

1. Hermansen F, Rosen SD, Fath-Ordoubadi F, Kooner JS, Clark JC, Camici PG, Lammertsma AA: Measurement of myocardial blood flow with oxygen-15 labelled water: comparison of different administration protocols. Eur J Nucl Med 1998, 25(7):751-759. *PDF*  
<http://jnm.snmjournals.org/content/39/10/1696.full.pdf>

## Cardiac Flow from Acetate PET

Van den Hoff et al. [1] have investigated and validated  $^{11}\text{C}$ -acetate as a flow tracer. This methodology is implemented as the **Card Acetate (1 Compartment)** model. It employs a single tissue compartment model



with tracer exchange between arterial plasma  $C_p(t)$  and myocardial tissue  $C_{myo}(t)$ .

## Operational Model Curve

The differential equation  $C_{myo}(t)$  is given by

$$\frac{dC_{myo}(t)}{dt} = K_1 C_p(t) - k_2 C_{myo}(t) = EF C_p(t) - k_2 C_{myo}(t)$$

$K_1$  is the product of flow  $F$  and extraction  $E$

$$K_1 = E \cdot F$$

For acetate, the extraction is flow dependent [1]

$$E(F) = 1 - 0.64e^{-1.2/F}$$

Furthermore a metabolite correction is necessary to derive the plasma activity from whole blood measured in the left cavity [1]

$$C_p = 0.91e^{-\ln 2(t/T_{1/2})} C_{LV}(t)$$

with  $T_{1/2}=5.3$  min. Including these relations into the differential equation yields

$$\frac{dC_{myo}(t)}{dt} = F(1 - 0.64e^{-1.2/F})0.91e^{-\ln 2(t/T_{1/2})} C_{LV}(t) - k_2 C_{myo}(t)$$

which is non-linear in  $F$ . The model curve incorporates a cardiac dual spillover correction, resulting in the operational equation

$$C_{Model}(t) = (1 - V_{LV} - V_{RV}) C_{myo}(t) + V_{RV} C_{LV}(t) + V_{RV} C_{RV}(t)$$

where

$V_{LV}$  = spill-over fraction of the blood activity in the left ventricular cavity  $C_{LV}(t)$ ,

$V_{RV}$  = spill-over fraction of the blood activity in the right ventricular cavity  $C_{RV}(t)$

## Implementation

When using the model from the PCARD module, the data are transferred appropriately.

When using it in PKIN the blood data have to be loaded as follows:

- ▶▶ The left ventricle curve must be loaded as the **Plasma activity** curve. No metabolite correction needs to be enabled on the **Blood** panel of PKIN because it is included in the tissue model.
- ▶▶ The right ventricle curve must be loaded as the **Whole blood** curve.

The following automatic adjustment is performed within the model:

- ▶▶ The spill-over fraction from the right ventricle  $V_{RV}$  is automatically fixed to zero if the string "Sep" is *not* contained in the name a region. The assumption is that such a TAC is not from septal tissue and should thus be modeled with spill-over from the left ventricle only.

## Parameter Fitting

The model includes the 4 fitable parameters **F**, **k2**, **vLV**, **vRV**. Please inspect the %SE standard error to get information about the reliability of the parameter estimates. The parameters for the correction of the flow-dependent extraction (**EF scale**, **EF exp**) and the metabolites (**MC scale**, **MC T12**) are initialized with the published values [1], but can be edited if needed. The **K1** parameter of the 1-tissue compartment model equivalent to the EF product is provided as a macro parameter.

## Reference

1. van den Hoff J, Burchert W, Borner AR, Fricke H, Kuhnel G, Meyer GJ, Otto D, Weckesser E, Wolpers HG, Knapp WH: [1-(11)C]Acetate as a quantitative perfusion tracer in myocardial PET. J Nucl Med 2001, 42(8):1174-1182. *PDF*  
<http://jnm.snmjournals.org/content/42/8/1174.full.pdf>

## Reference Models

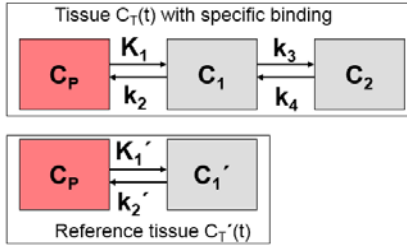
PKIN features the following list of reference models which are mostly used for the analysis of receptor ligand studies:

Model Name	Description
<i>4 Parameter Reference Tissue Model</i> (on page 143)	4 Parameter Reference Tissue Model by Lammertsma et al.
<i>SRTM</i> (on page 144)	Simplified Reference Tissue Model by Lammertsma and Hume
<i>SRTM2</i> (on page 145)	Modification of Simplified Reference Tissue Model by Wu and Carson to reduce BP bias induced by noise
<i>MRTM0</i> (on page 150)	Original Multilinear Reference Tissue Model by Ichise et al.
<i>MRTM</i> (on page 152)	Multilinear Reference Tissue Model by Ichise et al modified to reduce bias induced by noise
<i>MRTM2</i> (on page 154)	Multilinear Reference Tissue Model by Ichise et al modified to reduce bias and variability induced by noise
<i>Logan Non-Invasive</i> (on page 149)	Reference tissue method which is based on an average $k_2$
<i>2 Tissue Reference Model</i> (on page 156)	Watabe's reference tissue model with a 2-tissue compartment model in the reference region
<i>MP4A RLS (Nagatsuka)</i> (on page 159)	Multi-linear Reference Tissue Model for [11C]-MP4A (RLS)

## Full Reference Tissue Model

### Model Configuration and Assumptions

The **Full Reference Tissue** method (or *4 Parameter Reference Tissue Method*) of Lammertsma et al. is used for the analysis of studies with reversibly binding neuroreceptor tracers. A reference tissue devoid of receptors is required which can be modeled by a single-tissue compartment model. Tissues of interest which include the target receptor are modeled by a 2-tissue compartment model.



In the model equations  $C_T'(t)$  represents the tissue TAC from the reference region ( $k_3=0$  in the 2-tissue compartment model), and  $C_T(t)$  the tissue TAC from a receptor-rich region ( $k_3>0$ ). It is assumed that the distribution volume of the non-displaceable compartment is the same for tissues of interest and the reference tissue:  $K_1/k_2=K_1'/k_2'$ . Furthermore the reference tissue should not be affected by the pathology investigated under study.

### Operational Model Curve

Defining the ratio of tracer delivery  $R_1$  as  $K_1/K_1'$  and the binding potential  $BP_{ND}$  as  $k_3/k_4$ , the following operational equation can be derived for the measured tissue TAC in a receptor-rich region:

$$C_{Model}(t) = R_1 \left\{ C_T'(t) + aC_T'(t) \otimes e^{-ct} + bC_T'(t) \otimes e^{-dt} \right\}$$

The equation parameters and their relation to the model are given by:

$$\begin{aligned} a &= (k_3 + k_4 - c)(c - r) / p \\ b &= (d - k_3 - k_4)(d - r) / p \\ c &= (s + p) / 2 \\ d &= (s - p) / 2 \\ p &= \sqrt{s^2 - q} \\ q &= 4k_2k_4 \\ r &= k_2 / R_1 \\ s &= k_2 + k_3 + k_4 \end{aligned}$$

For convolution with the exponentials, the reference tissue TAC  $C_T'(t)$  is resampled on a regular grid, which can be specified by the **Resampling** parameter.

### Parameter Fitting

The operational equation includes four unknowns: **R1**, **k2**, **k3**, and **BP<sub>ND</sub>** (after substitution of  $BP_{ND}=k_3/k_4$ ), which can be fitted using nonlinear fitting techniques.



## Reference

Lammertsma AA, Bench CJ, Hume SP, Osman S, Gunn K, Brooks DJ, Frackowiak RS: Comparison of methods for analysis of clinical [ $^{11}\text{C}$ ]raclopride studies. *J Cereb Blood Flow Metab* 1996, 16(1):42-52. DOI <http://dx.doi.org/10.1097/00004647-199601000-00005>

## Simplified Reference Tissue Model (SRTM)

### Model Configuration and Assumptions

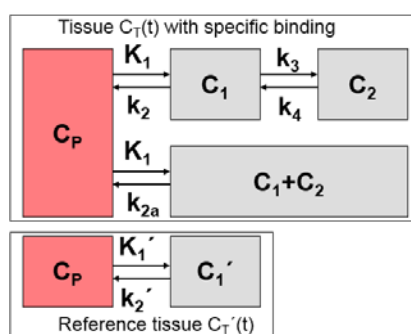
The **Simplified Reference Tissue Model** (SRTM) of Lammertsma and Hume [1] is used for the analysis of studies with reversibly binding neuroreceptor tracers. A reference tissue devoid of receptors is required which can be modeled by a single-tissue compartment model.

The assumptions of the model are:

- 1) The distribution volume is the same for the tissue of interest and the reference tissue:  
 $K_1/k_2 = K_1'/k_2'$ .

The kinetics in the receptor-rich tissue of interest is such that it is difficult to distinguish between the specific and the non-displaceable compartment; ie. the tissue TAC can be fitted by a 1-tissue compartment model with an uptake rate constant  $k_{2a} = k_2/(1+BP_{ND})$ .

Note that this assumption may not be valid for all tracers, and in this case SRTM calculates biased  $BP_{ND}$  estimates.



The model is a further development of the **4 Parameter Reference Tissue** (on page 143) method. It has been shown to be less dependent on the initial parameter values, more stable in the presence of noise, and provide more consistent estimates of relative tracer delivery  $R_1$ .

### Operational Model Curve

Defining the ratio of tracer delivery  $R_1$  as  $K_1/K_1'$  and the binding potential  $BP_{ND}$  as  $k_3/k_4$ , the following operational equation can be derived for the measured tissue TAC in a receptor-rich region:

$$C_{Model}(t) = R_1 C_T'(t) + [k_2 - R_1 k_2 / (1 + BP_{ND})] C_T'(t) \otimes e^{-k_2 t / (1 + BP_{ND})}$$

For convolution with the exponentials, the reference tissue TAC  $C'(t)$  is resampled on a regular grid, which can be specified by the **Resampling** parameter.

## Parameter Fitting

The operational equation includes four unknowns: **R1**, **k2**, and **BP<sub>nd</sub>**, which can be fitted using nonlinear fitting techniques. **k2'**, the transfer of tracer from the reference tissue back to the plasma, is provided as a derived parameter, as well as **k2a**.

**Note:** The reference methods MRTM2 and SRTM2 require  $k_2'$  as an input parameter. The  $k_2'$  resulting from the SRTM method is a suitable estimate. Therefore, when switching in PKIN from the SRTM model to the SRTM2 or MRTM2,  $k_2'$  is automatically copied from SRTM, in case the **Model conversion** option in the **Extras** panel is enabled.

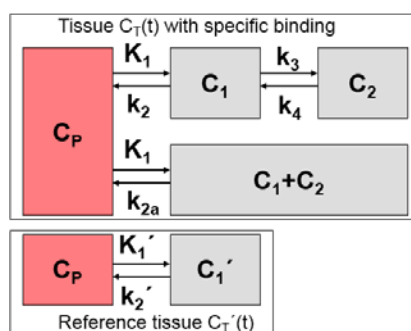
## Reference

1. Lammertsma AA, Hume SP: Simplified reference tissue model for PET receptor studies. *Neuroimage* 1996, 4(3 Pt 1):153-158. DOI <http://dx.doi.org/10.1006/nimg.1996.0066>

## Simplified Reference Tissue Model 2 (SRTM2)

### Model Configuration and Assumptions

The **Simplified Reference Tissue Model 2** (SRTM2) was derived by Wu and Carson [1] from the *SRTM* (on page 144) with the aim of improved parametric mapping. The model structure is the same



as well as the assumptions:

- 1) The distribution volume is the same for the tissue of interest and the reference tissue:  

$$K_1/k_2 = K_1'/k_2'$$

The kinetics in the receptor-rich tissue of interest is such, that it is difficult to distinguish between the specific and the free/non-specific compartment; ie. the TAC can be fitted by a 1-tissue compartment model with an uptake rate constant  $k_{2a} = k_2/(1+BP_{ND})$ .

## Operational Model Curve

The operational equation of the SRTM was re-written to allow for fixing of  $k_2'$ . This is relevant for parametric mapping because the model in each pixel TAC uses the same reference TAC and therefore should employ the same  $k_2'$ . Defining the ratio of tracer delivery  $R_1$  as  $K_1/K_1'$  and the binding potential  $BP_{ND}$  as  $k_3/k_4$ , the following operational equation can be derived for the measured TAC in a receptor-rich region:

$$C_{Model}(t) = R_1 C_T'(t) + R_1 [k_2' - k_{2a}] C_T'(t) \otimes e^{-k_{2a}t}$$

The three unknowns  $R_1$ ,  $k_2$  and  $k_{2a}$  in this equation can be fitted using nonlinear regression techniques. The binding potential can then be calculated as

$$BP_{ND} = R_1 \frac{k_2'}{k_{2a}} - 1.0$$

For convolution with the exponentials, the reference tissue TAC  $C_T'(t)$  is resampled on a regular grid, which can be specified by the **Resampling** parameter.

## Parameter Fitting

The operational equation includes three unknowns: **R1**, **k2**, and **k2a** which can be fitted using nonlinear fitting techniques. Note that **k2'**, the transfer of tracer from the reference tissue back to the plasma, can also be fitted. This can be employed by first estimating  $k_2'$  with a coupled fit and adequate TACs, and then fixing it for all regional fits as described *above* (on page 17). Furthermore, fixing of  $k_2'$  allows studying the bias introduced by an inadequate  $k_2'$ .

---

**Note:** As with the SRTM method,  $BP_{ND}$  estimates from SRTM2 tend to be biased if the 1-tissue compartment model assumption does not apply. The magnitude of the bias is even larger, most likely because the fixed  $k_2'$  can not compensate any more a part of the model inadequacy.

---

## Reference

1. Wu Y, Carson RE: Noise reduction in the simplified reference tissue model for neuroreceptor functional imaging. J Cereb Blood Flow Metab 2002, 22(12):1440-1452. DOI <http://dx.doi.org/10.1097/00004647-200212000-00004>

## Linear Simplified Reference Tissue Model (LSRTM)

This model was developed by Alpert et al. [1] to extend the concept of activation studies for including measurements targeting neurotransmitters and specific receptor populations. It uses the fact that cognitive activation increases neuronal firing rate, hereby increasing the endogenous neurotransmitter level and altering the kinetics of specifically bound radioligands. The methodology uses a single PET injection. The scan starts with an acquisition of the subject at rest (baseline), followed by an acquisition with the subject performing a task (activation). The data (after motion correction of the lengthy scan) is analyzed modeling the change of the endogenous neurotransmitter level as an effect which

occurs immediately after the task onset, and diminishes exponentially over time. The method has been applied to [11C]raclopride [1] and more recently to [18F]fallypride data [2,3].

Note: The model has been called LSSRM [1,2] and LSRRM [3], but was renamed to LSRTM to correspond to the commonly used notation of the Simplified Reference Tissue Model SRTM.

## Model Configuration and Assumptions

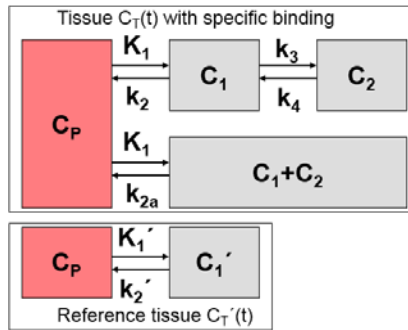
The model uses the same assumptions as the **Simplified Reference Tissue Model (SRTM)** (on page 144):

- 1) A reference tissue devoid of receptors exists which can be modeled by a single-tissue compartment model.

The distribution volume is the same for the tissue of interest and the reference tissue:

$$K_1/k_2 = K_1'/k_2'.$$

The kinetics in the receptor-rich tissue of interest is such that it is difficult to distinguish between the specific and the non-displaceable compartment; ie. the tissue TAC can be fitted by a 1-tissue compartment model with an apparent uptake rate constant  $k_{2a} = k_2/(1+BP_{ND})$ .



Additionally, it is assumed that at the onset of the activation,  $k_{2a}$  is increased by the amplitude  $\gamma$  of ligand displacement, and that the effect dies away with a decay constant  $\tau$ . This behavior is described by a time-dependent  $k_{2a}$  as follows:

$$k_{2a}(t) = k_{2a} + \gamma h(t)$$

$$h(t) = \begin{cases} 0 & t < \text{Begin} \\ e^{-\tau u} & t \geq \text{Begin}, u = t - \text{Begin} \end{cases}$$

## Operational Model Curve

Defining the ratio of tracer delivery  $R_1$  as  $K_1/K_1'$ , the following operational equation can be derived for the measured tissue TAC in a receptor-rich region:

$$C_{Model}(t) = R_1 C_T'(t) + k_2 \int_0^t C_T'(s) ds - k_{2a} \int_0^t C_T(s) ds - \gamma \int_0^t h(s) C_T(s) ds$$

From the parameters in the equation the binding potential  $BP_{ND}$  can be calculated as

$$BP_{ND} = \frac{k_2}{k_{2a}} - 1.0$$

The main outcome of the model, however, is the amplitude  $\gamma$  of ligand displacement, for which a t-score value and its p-value for a one-tailed t-test are also provided.

### Parameter Fitting

The user has to define the **Activation** start time with the corresponding input parameter. The **Resampling** parameter defines the interpolation grid for the exponential convolution, which corresponds to the last integral term in the operational equation.

The parameters of the non-linear fitting procedure include **R1**, **k2**, **k2a**, **gamma** and **tau**. To reduce the number of fitted parameters **tau** is usually fixed at a value of  $0.03\text{min}^{-1}$  (~23min half-life). From the fit parameters, **BPnd** is calculated as well as the **t-score** and the **p-value** of **gamma**.

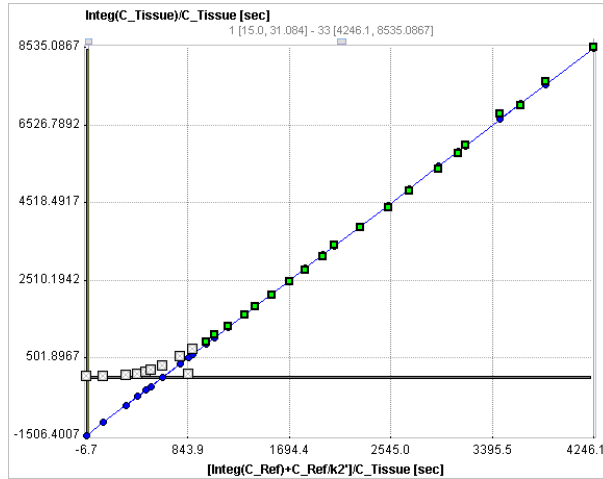
### References

- [1] Alpert NM, Badgaiyan RD, Livni E, Fischman AJ. A novel method for noninvasive detection of neuromodulatory changes in specific neurotransmitter systems. *Neuroimage*. 2003;19(3):1049-60. DOI [http://dx.doi.org/10.1016/S1053-8119\(03\)00186-1](http://dx.doi.org/10.1016/S1053-8119(03)00186-1)
- [2] Christian BT, Lehrer DS, Shi B, Narayanan TK, Strohmeyer PS, Buchsbaum MS, Mantil JC. Measuring dopamine neuromodulation in the thalamus: using [F-18]fallypride PET to study dopamine release during a spatial attention task. *Neuroimage*. 2006;31(1):139-52. DOI
- [3] Ceccarini J, Vrieze E, Koole M, Muylle T, Bormans G, Claes S, Van Laere K. Optimized in vivo detection of dopamine release using 18F-fallypride PET. *J Nucl Med*. 2012;53(10):1565-72. DOI

## Logan's Reference Tissue Model based on Average k<sub>2</sub>'

### Model Configuration and Assumptions

Logan et al. [1] developed a reference tissue method for reversible receptor ligands which does not depend on a specific model structure of the reference tissue. Assuming the presence of reference region TAC  $C_T'(t)$  with an average tissue-to-plasma clearance  $k_2'$ , the target tissue TAC  $C_T(t)$  is transformed and plotted as a function of the transformed reference TAC, as illustrated below.



The operational equation resembles a linear equation with the distribution volume ratio ( $DVR = BP_{ND} + 1$ ) as the slope plus an error term which decreases over time. Therefore the late part starting from a time  $t^*$  of the plotted samples can be fitted by a regression line and the slope used for calculating  $BP_{ND}$ . The time  $t^*$  can be determined as the time after which no further significant increases in slope are observed.

### Operational Model Curve

The graphical plot of the **Logan Reference Tissue** method is described by the following equation with the form resembling a linear equation.

$$\frac{\int_0^t C_T(\tau) d\tau}{C_T(t)} = DVR \left[ \frac{\int_0^t C_T'(\tau) d\tau + C_T'(t)/k_2'}{C_T(t)} \right] + b = (BP_{ND} + 1) \left[ \frac{\int_0^t C_T'(\tau) d\tau + C_T'(t)/k_2'}{C_T(t)} \right] + b$$

$k_2'$  in the original publication was the population average  $k_2$  determined for the reference tissue using blood sampling, but using the subject's own  $k_2'$  may be preferable.

### Parameter Fitting

After switching to the **Logan Reference** model in PKIN a suitable reference region must be selected. **k<sub>2</sub>'** is an input parameter which must be manually entered, or estimated by using other models such as the MRTM or SRTM model. **Logan Reference** fits a linear regression to the tail of the plot starting at time  $t^*$ . Note that  $t^*$  is specified in regular acquisition time,

which is translated into the "funny time" of the x-axis and shown as the **Start** parameter. The result parameter of interest is the **BPnd** which is calculated from the slope.

A reasonable value of  $t^*$  can be estimated by the based on the error criterion **Max Err.** For instance, if **Max Err.** is set to 10% and the fit box of  $t^*$  is checked, the model searches the earliest sample such that the deviation between the regression and all measurements is less than 10%. Samples earlier than the  $t^*$  time are disregarded for regression and thus painted in gray. In order to apply the analysis to the same data segment in all regions, please switch off the fit box of  $t^*$ , propagate the model with the **Copy to all Regions** button, and then activate **Fit all regions**.

---

**Note:** The  $k_2'$  resulting from the SRTM or MRTM method might be a reasonable estimate for the average clearance rate  $k_2'$ . Therefore, when switching in PKIN from the SRTM or MRTM model to Logan Reference model,  $k_2'$  of those methods is automatically copied to  $k_2'$ , in case **Model conversion** in the **Configuration** panel is enabled. See *above* (on page 17) for recommendations how to estimate  $k_2'$ .

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## Reference

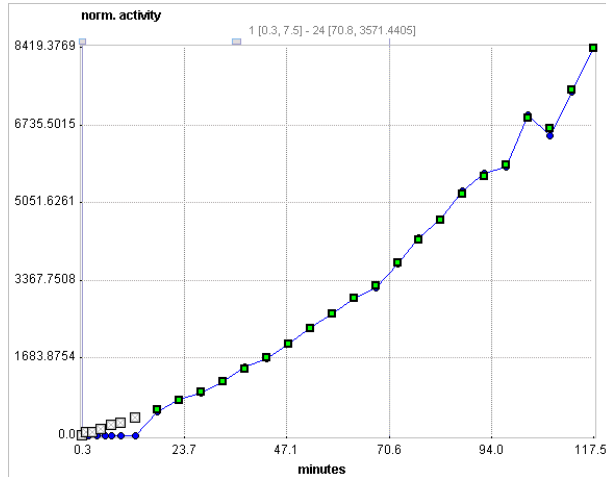
1. Logan J, Fowler JS, Volkow ND, Wang GJ, Ding YS, Alexoff DL: Distribution volume ratios without blood sampling from graphical analysis of PET data. J Cereb Blood Flow Metab 1996, 16(5):834-840. DOI <http://dx.doi.org/10.1097/00004647-199609000-00008>

## Ichise's Original Multilinear Reference Tissue Model (MRTM0)

### Model Configuration and Assumptions

Starting from the operational equation of the blood-based *Logan plot* (on page 166), Ichise et al. derived three multi-linear reference tissue model variants MRTM0, MRTM and MRTM2 [1,2]. They all assume an initial equilibration time  $t^*$  from which on the derived multi-linear relation holds. However, if kinetics in the target tissue can be described by a 1-tissue compartment model (an assumption required for the SRTM), all data can be used for the fitting ( $t^*=0$ ). Otherwise an adequate  $t^*$  value has to be determined.

Assuming the presence of receptor-devoid reference region TAC  $C_T'(t)$ , the target tissue TAC  $C_T(t)$  is transformed and plotted as a function of the transformed reference TAC, as illustrated below. For the calculation of  $BP_{ND}$  it is assumed that the non-displaceable distribution volumes in the tissue and reference regions are identical.



## Operational Model Curve

The **MRTM0** model curve is described by

$$\frac{\int_0^t C_T(\tau) d\tau}{C_T(t)} = \frac{V_T}{V_T'} \frac{\int_0^t C_T'(\tau) d\tau}{C_T(t)} + \frac{V_T}{V_T' k'_2} \frac{C_T'(t)}{C_T(t)} + b$$

where  $V_T$  and  $V_T'$  are the total distribution volumes of  $C_T(t)$  and  $C_T'(t)$ ,  $k'_2$  is the clearance rate constant from the reference region to plasma, and  $b$  is the intercept term, which becomes constant for  $T > t^*$ . The multi-linear relationship above can be fitted using multi-linear regression, yielding three regression coefficients. From the first coefficient the binding potential can be calculated by

$$BP_{ND} = \frac{V_T}{V_T'} - 1.0 = DVR - 1$$

## Parameter Fitting

After switching to the **Ichise NonInvasive MRTM0** model a suitable reference region must be selected. It allows fitting a multi-linear regression to the operational equation within a range defined by the parameters **t\*** and **End Lin**. The results are the three regression coefficients, and the derived binding potential **BPnd**.

A reasonable value of **t\*** can be estimated by the based on the error criterion **Max Err**. For instance, if **Max Err** is set to 10% and the fit box of **t\*** is checked, the model searches the earliest sample such that the deviation between the regression and all measurements is less than 10%. Samples earlier than the **t\*** time are disregarded for regression and thus painted in gray. In order to apply the analysis to the same data segment in all regions, please switch off the fit box of **t\***, propagate the model with the **Copy to all Regions** button, and then activate **Fit all regions**.



For receptor ligands with 1-tissue kinetics such as [11C]DASB the multi-linear equation is correct from  $t^*=0$ , and  $b$  is equal to  $(-1/k_2)$ , where  $k_2$  is the clearance rate constant from the tissue to plasma. Furthermore,  $R_1 = K_1/K'_1$ , the relative radioligand delivery, can be calculated from the ratio of the second and third regression coefficients.

## References

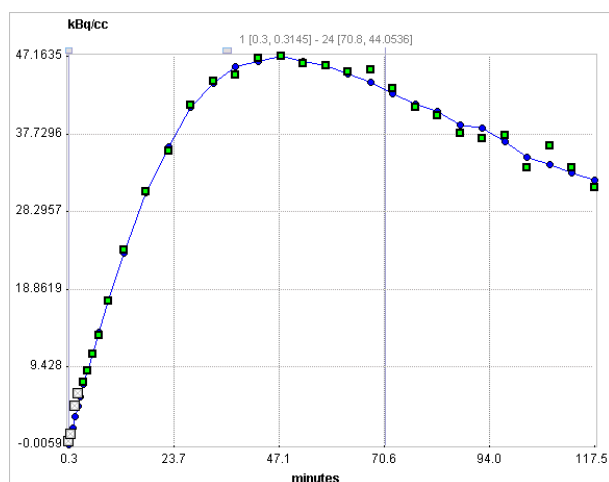
1. MRTM0: Ichise M, Ballinger JR, Golan H, Vines D, Luong A, Tsai S, Kung HF: Noninvasive quantification of dopamine D2 receptors with iodine-123-IBF SPECT. J Nucl Med 1996, 37(3):513-520.
2. Comparison of the MRTM and SRTM models: Ichise M, Liow JS, Lu JQ, Takano A, Model K, Toyama H, Suhara T, Suzuki K, Innis RB, Carson RE: Linearized reference tissue parametric imaging methods: application to [11C]DASB positron emission tomography studies of the serotonin transporter in human brain. J Cereb Blood Flow Metab 2003, 23(9):1096-1112. DOI <http://dx.doi.org/10.1097/01.WCB.0000085441.37552.CA>

## Ichise's Multilinear Reference Tissue Model (MRTM)

### Model Configuration and Assumptions

Starting from the operational equation of the blood-based *Logan plot* (on page 166), Ichise et al. derived three multi-linear reference tissue model variants MRTM0, MRTM and MRTM2 [1]. They all assume an initial equilibration time  $t^*$  from which on the derived multi-linear relation holds. However, if kinetics in the target tissue can be described by a 1-tissue compartment model (an assumption required for the SRTM), all data can be used for the fitting ( $t^*=0$ ). Otherwise an adequate  $t^*$  value has to be determined.

Assuming the presence of receptor-devoid reference region TAC  $C_T'(t)$ , the target tissue TAC  $C_T(t)$  is plotted as a function of the transformed tissue TACs as illustrated below. For the calculation of  $BP_{ND}$  it is assumed that the non-displaceable distribution volumes in the tissue and reference regions are identical.



## Operational Model Curve

To reduce noise-related bias effects arising in the MRTM0 method Ichise et al. applied a strategy known to be effective in reducing the noise-induced bias for the models requiring blood data. To this end the equation of the **MRTM0** (on page 150) method was rearranged to remove the noisy tissue radioactivity term  $C_T(t)$  from the independent variables. This approach resulted in a new method called MRTM with following operational equation for  $C_T(t)$ :

$$C_{Model}(t) = -\frac{V_T}{V_T' b} \int_0^t C_T'(\tau) d\tau + \frac{1}{b} \int_0^t C_T(\tau) d\tau - \frac{V_T}{V_T' k_2' b} C_T'(t)$$

The multi-linear relationship above can be fitted using multi-linear regression, yielding three regression coefficients. The binding potential can then be calculated by dividing the first two regression coefficients

$$BP_{ND} = \frac{V_T}{V_T'} - 1.0 = DVR - 1$$

Furthermore, division of the first by the third regression coefficient yields an estimate of  $k_2'$ .

## Parameter Fitting

After switching to the **Ichise NonInvasive MRTM** model a suitable reference region must be selected. It allows fitting the multi-linear operational equation to the tissue TAC using the data segment starting from  $t^*$ . The results are the three regression coefficients, and the binding potential **BPnd** and **k2'**.

A reasonable value of  $t^*$  can be estimated by the based on the error criterion **Max Err**. For instance, if **Max Err**. is set to 10% and the fit box of  $t^*$  is checked, the model searches the earliest sample such that the deviation between the regression and all measurements is less than 10%. Samples earlier than the  $t^*$  time are disregarded for regression and thus painted in gray. In order to apply the analysis to the same data segment in all regions, please switch off the fit box of  $t^*$ , propagate the model with the **Copy to all Regions** button, and then activate **Fit all regions**.

For receptor ligands with 1-tissue kinetics such as [11C]DASB the multi-linear equation is correct from  $t^*=0$ , and the clearance rate constant from the tissue to plasma  $k_2$  is equal to the negative value of the second regression coefficient,  $-(1/b)$ . Furthermore,  $R_1 = K_1/K_1'$ , the relative radioligand delivery, equals the third regression coefficient.

---

**Note:** The reference methods MRTM2 and SRTM2 require  $k_2'$  as an input parameter. The  $k_2'$  resulting from the MRTM method above might be a suitable estimate. Therefore, when switching in PKIN from the MRTM model to MRTM2 or SRTM2,  $k_2'$  is automatically copied from MRTM, if the **Model conversion** option in the **Extras** panel is enabled.

---

## Reference

1. Ichise M, Liow JS, Lu JQ, Takano A, Model K, Toyama H, Suhara T, Suzuki K, Innis RB, Carson RE: Linearized reference tissue parametric imaging methods: application to

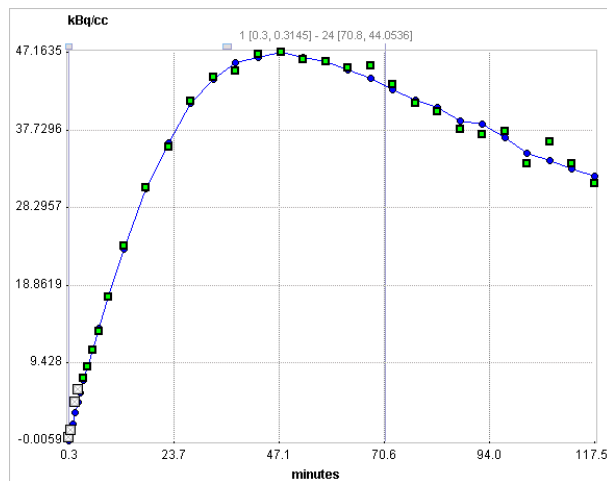
[11C]DASB positron emission tomography studies of the serotonin transporter in human brain. J Cereb Blood Flow Metab 2003, 23(9):1096-1112. DOI <http://dx.doi.org/10.1097/01.WCB.0000085441.37552.CA>

## Ichise's Multilinear Reference Tissue Model 2 (MRTM2)

### Model Configuration and Assumptions

Starting from the operational equation of the blood-based *Logan plot* (on page 166), Ichise et al. derived three multi-linear reference tissue model variants MRTM0, MRTM and MRTM2 [1]. They all assume an initial equilibration time  $t^*$  from which on the derived multi-linear relation holds. However, if kinetics in the target tissue can be described by a 1-tissue compartment model (an assumption required for the SRTM), all data can be used for the fitting ( $t^*=0$ ). Otherwise an adequate  $t^*$  value has to be determined.

Assuming the presence of receptor-devoid reference region TAC  $C_T'(t)$ , the target tissue TAC  $C_T(t)$  is plotted as a function of the transformed tissue TACs as illustrated below. For the calculation of  $BP_{ND}$  it is assumed that the non-displaceable distribution volumes in the tissue and reference regions are identical.



### Operational Model Curve

When applied to noisy data such as single-pixel TACs in parametric mapping, the MRTM method still suffers from a high variability. Assuming a known value of the reference tissue clearance rate  $k_2'$  the MRTM operational equation can be reformulated as the MRTM2 operational equation:

$$C_{Model}(t) = -\frac{V_T}{V_T' b} \left( \int_0^t C_T'(\tau) d\tau + \frac{1}{k_2'} C_T'(t) \right) + \frac{1}{b} \int_0^t C_T(\tau) d\tau$$

with only two regression coefficients  $V_T/(V_T'b)$  and  $1/b$  for  $T > t^*$ . The multi-linear relationship above can be fitted using multi-linear regression, yielding three regression coefficients. The binding potential is then calculated from the ratio of the two regression coefficients as

$$BP_{ND} = \frac{V_T}{V_T'} - 1.0 = DVR - 1$$

For receptor ligands with 1-tissue kinetics such as [11C]DASB the multi-linear equation is correct from  $t^*=0$ , and the clearance rate constant from the tissue to plasma  $k_2$  is equal to the negative value of the second regression coefficient,  $-(1/b)$ . Furthermore,  $R_1 = K_1/K_1'$ , the relative radioligand delivery, equals the first regression coefficient divided by  $k_2'$ .

### Parameter Fitting

After switching to the **Ichise NonInvasive MRTM2** model a suitable reference region must be selected. It allows fitting the multi-linear operational equation to the tissue TAC using the data segment starting from  $t^*$  and a fixed value of  $k_2'$  which needs to be specified. The results are the two regression coefficients, and the binding potential **BPnd**.

A reasonable value of  $t^*$  can be estimated based on the error criterion **Max Err**. For instance, if **Max Err** is set to 10% and the fit box of  $t^*$  is checked, the model searches the earliest sample such that the deviation between the regression and all measurements is less than 10%. Samples earlier than the  $t^*$  time are disregarded for regression and thus painted in gray. In order to apply the analysis to the same data segment in all regions, please switch off the fit box of  $t^*$ , propagate the model with the **Copy to all Regions** button, and then activate **Fit all regions**.

---

**Note:** The  $k_2'$  resulting from the SRTM or MRTM method may be a suitable estimate for use in MRTM2. Therefore, when switching in PKIN from the SRTM or MRTM model to MRTM2,  $k_2'$  is automatically copied, if the **Model conversion** option in the **Extras** panel is enabled. Further suggestions how to obtain an adequate  $k_2'$  value are given *above* (on page 17).

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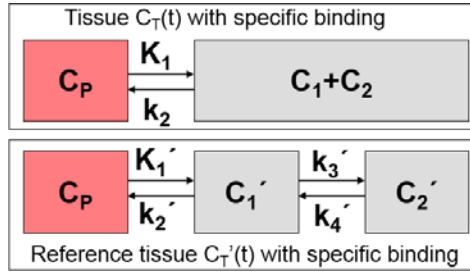
### Reference

1. Ichise M, Liow JS, Lu JQ, Takano A, Model K, Toyama H, Suhara T, Suzuki K, Innis RB, Carson RE: Linearized reference tissue parametric imaging methods: application to [11C]DASB positron emission tomography studies of the serotonin transporter in human brain. J Cereb Blood Flow Metab 2003, 23(9):1096-1112. DOI <http://dx.doi.org/10.1097/01.WCB.0000085441.37552.CA>

## Watabe's Reference Tissue Model with 2 Compartments

### Model Configuration and Assumptions

The **2 Tissue Reference Tissue** method proposed by Watabe [1] has been developed for ligands which do not follow the usual assumptions of the reference tissue models. Instead of a 1-tissue compartment model, the reference tissue is described by a 2-tissue model as illustrated below. The second compartment  $C_2'$  may represent specific or non-specific binding.



### Operational Model Curve

Assuming that the non-specific distribution volume of both tissues is the same, the following operational equation for fitting the tissue TAC can be derived (see Millet et al [2], appendix C):

$$C_{Model}(t) = R_1 \left[ C_T'(t) + a C_T'(t) \otimes e^{-(k_1' + k_4')t} + b C_T'(t) \otimes e^{-k_2 t} \right]$$

When the parameters  $k_2'$ ,  $k_3'$ , and  $k_4'$  are fixed to constants determined with receptor-rich tissue, only  $R_1$  and  $k_2$  need to be fitted for estimating the binding potential  $BP_{ND}$ . The equation parameters and their relation to the model are given by:

$$a = \frac{k_3' k_2'}{k_3' + k_4' - k_2}$$

$$b = \frac{k_2^2 - k_2(k_2' + k_3' + k_4') + k_2' k_4'}{k_3' + k_4' - k_2 + k_2^2}$$

$$BP_{ND} = \frac{V_T}{V_T'} - 1 = \frac{K_1 k_2'}{K_1' k_2 (1 + k_3' / k_4')} - 1$$

For convolution with the exponentials, the reference tissue TAC  $C_T'(t)$  is resampled on a regular grid, which can be specified by the **Resampling** parameter.

### Parameter Fitting

After switching to the **2 Tissue Reference Model** a suitable reference region must be selected. The operational equation includes six fitting parameters: **R1**, **k2**, **k2'**, **k3'**, **k4'**. Per default, the fit boxes of **k2'**, **k3'** and **k4'** are off with the idea, that the values are determined externally. However, any of them may also be fitted. Fitting is performed using nonlinear techniques, and the main outcome is **BPnd**.

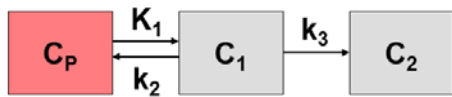
## References

[1] Original abstract: Watabe, H et al. The reference tissue model: Three compartments for the reference region. *Neuroimage* 11: S12.

[2] Model description in Appendix C: Millet P, Graf C, Buck A, Walder B, Ibanez V: Evaluation of the reference tissue models for PET and SPECT benzodiazepine binding parameters. *Neuroimage* 2002, 17(2):928-942. DOI <http://dx.doi.org/10.1006/nimg.2002.1233>

## Patlak Reference Tissue Model

The Patlak plot has been developed by Patlak and Blasberg [1] for tracers undergoing irreversible trapping. Most often it is applied for the analysis of FDG, which can be modeled as a 2-tissue compartment model with  $k_4=0$ .



However, this model structure is not necessary for the application of the method. It is sufficient to have any compartment in the system which binds irreversibly.

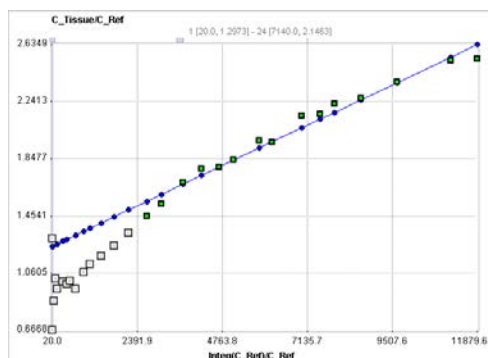
When the plasma activity is not available, the Patlak plot can be employed as a reference method provided that there exists some tissue wherein tracer is not irreversibly trapped. The procedure simply replaces the input curve by the reference tissue TAC.

## Operational Model Curve

The Patlak plot belongs to a group of *Graphical Analysis* techniques, whereby the measured tissue TAC  $C_T(t)$  undergoes a mathematical transformation and is plotted against some sort of "normalized time". The Patlak plot using reference tissue is given by the expression

$$\frac{C_T(t)}{C_T'(t)} = K \frac{\int_0^t C_T'(\tau) d\tau}{C_T'(t)} + V$$

with the reference tissue TAC  $C_T'(t)$ . This means that the measured PET activity is divided by the reference tissue activity, and plotted at a "normalized time" (integral of the reference TAC from the injection time divided by the instantaneous reference activity). For systems with irreversible compartments this plot will result in a straight line after an equilibration time  $t^*$ .



Under several assumptions, including a common  $K_1/k_2$ , the slope of the linear regression represents the following relation

$$\text{slope} = K = \frac{k_2 k_3}{(k_2 + k_3)(1 - K_{eq})}$$

with the equilibrium constant  $K_{eq}$ .

The reference Patlak plot has been applied for the FDOPA PET tracer for calculating an index of the influx  $K_i$ . Both the cerebellum and the occipital lobe have been used as the reference [2].

## Parameter Fitting

The **Patlak Reference** model calculates and displays the transformed measurements as described by the formula above. It allows fitting a regression line to the data segment starting at time  $t^*$ . The results are the slope **K** and the **Intercept** of the regression line.

$t^*$  can be specified manually, or a value estimated using an error criterion **Max Err**. For instance, if **Max Err** is set to 10% and the fit box of  $t^*$  is checked, the model searches the earliest sample so that the deviation between the regression line and all measurements is less than 10%. Samples earlier than the  $t^*$  time are disregarded for regression and thus painted in gray. Note that  $t^*$  must be specified in real acquisition time, although the x-axis units are in "normalized time". The corresponding normalized time which can be looked up in the plot is shown as a non-fitable result parameter **Start**. In order to apply the analysis to the same data segment in all regions, please switch off the fit box of  $t^*$ , propagate the model with the **Copy to all Regions** button, and then activate **Fit all regions**.

## References:

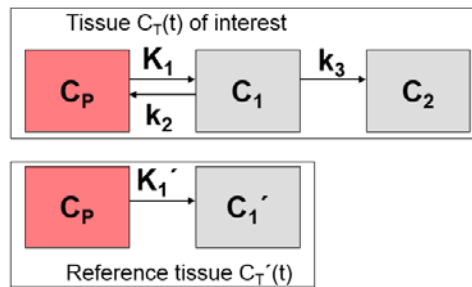
- [1] Patlak CS, Blasberg RG: Graphical evaluation of blood-to-brain transfer constants from multiple-time uptake data. Generalizations. J Cereb Blood Flow Metab 1985, 5(4):584-590. DOI <http://dx.doi.org/10.1038/jcbfm.1985.87>

[2] Sossi V, Holden JE, de la Fuente-Fernandez R, Ruth TJ, Stoessl AJ: Effect of dopamine loss and the metabolite 3-O-methyl-[18F]fluoro-dopa on the relation between the 18F-fluorodopa tissue input uptake rate constant  $K_{occ}$  and the [18F]fluorodopa plasma input uptake rate constant  $K_i$ . J Cereb Blood Flow Metab 2003, 23(3):301-309. DOI

## Multi-linear Reference Tissue Model for [11C]-MP4A (RLS)

### Model Configuration and Assumptions

The **MP4A RLS (Nagatsuka)** model has been developed for the non-invasive quantification method (RLS) of the acetylcholinesterase (AChE) activity in the human brain from measurements with the  $^{11}\text{C}$ -MP4A acetylcholine analog [1]. In contrast to reference methods for receptor tracers which use a reference devoid of specific binding, the present method uses a reference with very high AChE activity which immediately traps the tracer so there is no washout.  $C_T(t)$  is the TAC from a cortical target region, and  $C_T'(t)$  the TAC from the reference region (striatum or cerebellum).  $k_3$  represents the rate of tracer hydrolysis by AChE.



### Operational Model Curve

By applying the method of Blomqvist, the following multi-linear equation is derived

$$C_{Model}(t) = p_1 C_T'(t) + p_2 \int_0^t C_T'(\tau) d\tau + p_3 \int_0^t C_T(\tau) d\tau$$

It can be fitted to  $C_T(t)$  using multi-linear regression, yielding three regression coefficients from which three parameters of interest can be calculated:

$$\begin{aligned} R_1 &= K_1 / K_1' = p_1 \\ k_2 &= -(p_3 + p_2 / p_1) \\ k_3 &= p_2 / p_1 \end{aligned}$$

### Parameter Fitting

After switching to the **MP4A RLS (Nagatsuka)** model a suitable reference region must be selected. The findings in different publications indicate that cerebellum yields more stable results than striatum, most likely due to the higher impact of motion on the signal from the small striatum than the large cerebellum.

While the regression coefficients represent the fitting parameters, the resulting rate constants are shown in the derived parameter section.



## Reference

1. Nagatsuka Si S, Fukushi K, Shinotoh H, Namba H, Iyo M, Tanaka N, Aotsuka A, Ota T, Tanada S, Irie T: Kinetic analysis of [(11)C]MP4A using a high-radioactivity brain region that represents an integrated input function for measurement of cerebral acetylcholinesterase activity without arterial blood sampling. J Cereb Blood Flow Metab 2001, 21(11):1354-1366.  
*DOI <http://dx.doi.org/10.1097/00004647-200111000-00011>*

## Non-Compartmental Models

PKIN includes some additional models, which are neither explicit compartment nor reference models. They include the important linear regression methods **Logan Plot** and **Patlak Plot**, recent alternative methods for estimating the total distribution volume, as well as simple methods to quickly calculate ratios between tissue TACs and/or the plasma concentration.

Model Name	Description
<i>Logan Plot</i> (on page 166)	Graphical analysis of reversible system to calculate the total distribution volume.
<i>Ichise MA1</i> (on page 172)	Reformulation of the Logan plot as a multilinear analysis to reduce noise-induced bias. Result is the total distribution volume.
<i>Ichise MA2</i> (on page 173)	Multilinear analysis derived from the two-compartment model. Results are the total as well as the specific distribution volume.
<i>Patlak Plot</i> (on page 163)	Graphical analysis of irreversible system. Applied to FDG studies it allows to estimate glucose turnover.
Bolus/Infusion Optimization	Based on a bolus study, the bolus/infusion ratio for an equilibrium study can be estimated
<i>Area under Curve</i> (on page 179)	Calculates the integrals of the tissue TAC and the input curve, as well as their ratio.
Tissue/Plasma Ratio	Calculate the ratio of tissue binding to plasma concentration in order to get an estimate of the total DV
<i>Ratio Methods</i> (on page 181)	Calculate the ratio of specific binding to non-specific binding.
<i>Retention Fraction</i> (on page 182)	Calculate the fraction of passed tracer which is accumulated in tissue
<i>Fractal Dimension</i> (on page 183)	Calculation of the box-counting dimension of curves

## Spectral Analysis

The **Spectral** model allows performing a *regular Spectral Analysis (SA)* (on page 20) given a TAC and an input curve according to Cunningham and Jones [1].

## Operational Model Curve

The operational equation of SA is given by

$$C_{PET}(t) = \sum_{i=1}^N a_i C_p(t) \otimes e^{-\beta_i t} = \sum_{i=1}^N a_i B_i(t)$$

that is, tissue uptake is modeled as a sum of N possible tissue responses. Due to the constraint of first order tracer kinetics, the coefficients  $a_i$  and the decay constants  $\beta_i$  must be non-negative. In practice, a discrete set of the decay constants  $\beta_i$  is selected which covers the physiologically reasonable range, typically logarithmically spaced in the range  $[10^{-5}, 1]\text{sec}^{-1}$ . The corresponding tissue responses

$$B_i(t) = C_p(t) \otimes e^{-\beta_i t}$$

are the *Basis Functions* of spectral analysis. When fitting the operational equation above to a tissue TAC, the only unknowns are the coefficients  $a_i$ , because only a pre-defined set of discrete  $\beta_i$  values is considered. Therefore, the problem is that of a non-negative linear least squares estimation (NNLS) with the constraint of non-negative coefficients.

An advantage of SA is the fact that no particular compartment structure is imposed. Rather, its result can be used to estimate how many kinetic tissue compartments can be resolved by PET. To this end, the results are plotted as a spectrum with the selected decay constants  $\beta_i$  along the x-axis (as the "frequencies") and the estimated coefficients  $a_i$  along the y-axis (as the "amplitudes"). Because of the large range,  $\log(\beta_i)$  is used in spectrum plotting rather than  $\beta_i$ . The number of peaks in this spectrum corresponds to the number of distinct compartments. A peak appearing to the far left (low frequency, slow component) indicates irreversible trapping. A peak to the far right (high frequency, fast component) corresponds to kinetics indistinguishable from the input curve, thus to vascular contributions. Intermediate peaks represent compartments which exchange reversibly with plasma or with other tissue compartments.

## Parameter Fitting

In the **Spectral** model **#Exponentials** determines the number of discrete  $\beta_i$  decay constants which are logarithmically spaced in the range between **Beta-min** and the **Beta-max**. There is no well-defined minimum value as in the case of decay-corrected TACs where it is equal to the decay constant of the isotope.

Estimates of the tissue uptake constant **K1** and the total distribution volume **Vt** can be calculated based on the peaks found in the spectrum. Since a blood-related peak distorts these estimates, it can be excluded from these calculations by the **Exclude HF** check box.

The SA results can be visualized in 3 different ways with a corresponding radio button setting:

- 1) **Display TAC** shows the TAC together with the model curve calculated from the detected peaks.

**Display Spectrum** shows the peaks in the spectrum.

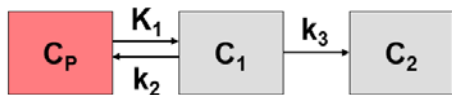
**Display unit IRF** shows the Impulse Response Function which depends on the **Exclude HF** setting.

## Reference

Cunningham VJ, Jones T: Spectral analysis of dynamic PET studies. J Cereb Blood Flow Metab 1993, 13(1):15-23.

## Patlak Plot

The Patlak plot has been developed by Patlak et al. [1] for tracers undergoing irreversible trapping. Most often it is applied for the analysis of FDG, which can be modeled as a 2-tissue compartment model with  $k_4=0$ .



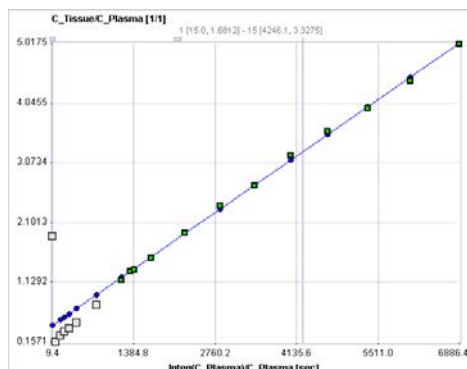
However, this model structure is not necessary for the application of the method. It is sufficient to have any compartment in the system which binds irreversibly.

## Operational Model Curve

The Patlak plot belongs to a group of *Graphical Analysis* techniques, whereby the measured tissue TAC  $C_T(t)$  undergoes a mathematical transformation and is plotted against some sort of "normalized time". The Patlak plot is given by the expression

$$\frac{C_T(t)}{C_P(t)} = K \frac{\int_0^t C_P(\tau) d\tau}{C_P(t)} + V$$

with the input curve  $C_P(t)$ . This means that the measured PET activity is divided by plasma activity, and plotted at a "normalized time" (integral of the input curve from the injection time divided by the instantaneous plasma activity). For systems with irreversible compartments this plot will result in a straight line after an equilibration time  $t^*$ .



The slope  $K$  and the intercept  $V$  must be interpreted according to the underlying compartment model. For the FDG model mentioned, the slope  $K$  equals  $K_1 k_3 / (k_2 + k_3)$  and represents the metabolic flux, while the intercept  $V$  equals  $V_0 + vB$  with the distribution volume  $V_0$  of the reversible compartment  $C_1$  and the fractional blood volume  $vB$ .

## Parameter Fitting

The **Patlak Plot** model calculates and displays the transformed measurements as described by the formula above. It allows fitting a regression line to the data segment starting at time  $t^*$ . The results are the **Slope** and the **Intercept** of the regression line.

$t^*$  can be specified manually, or a value estimated using an error criterion **Max Err**. For instance, if **Max Err** is set to 10% and the fit box of  $t^*$  is checked, the model searches the earliest sample so that the deviation between the regression line and all measurements is less than 10%. Samples earlier than the  $t^*$  time are disregarded for regression and thus painted in gray. Note that  $t^*$  must be specified in real acquisition time, although the x-axis units are in "normalized time". The corresponding normalized time which can be looked up in the plot is shown as a non-fitable result parameter **Start**. In order to apply the analysis to the same data segment in all regions, please switch off the fit box of  $t^*$ , propagate the model with the **Copy to all Regions** button, and then activate **Fit all regions**.

For the analysis of FDG data, the **Lumped Constant** (LC) and the **Plasma glucose** level (PG) of the patient should be entered. The metabolic rate of glucose **MRGlu** is then obtained from the regression slope by

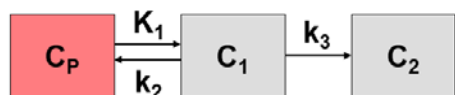
$$MRGlu = slope \frac{PG}{LC}$$

## Reference:

1. Patlak CS, Blasberg RG, Fenstermacher JD: Graphical evaluation of blood-to-brain transfer constants from multiple-time uptake data. J Cereb Blood Flow Metab 1983, 3(1):1-7. DOI <http://dx.doi.org/10.1038/jcbfm.1983.1>

## MLAIR (Multiple Linear Analysis for Irreversible Radiotracers)

The MLAIR method has been developed by Kim et al. [1] for tracers undergoing irreversible trapping. It is an alternative to the *Patlak plot* (on page 163) analysis which is dependent on the specification of an equilibration time  $t^*$  which may vary among tissues, and may suffer from bias when applied to noisy data. It has applied for the analysis of FDG, which can be modeled as a 2-tissue compartment model with  $k_4=0$ , and  $^{11}\text{C}$ -labeled MeNTI [1].



## Operational Model Curve

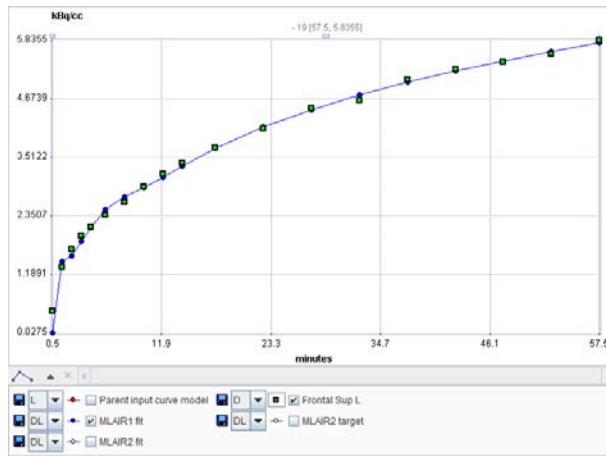
The authors develop two approaches, both of which are included in the **Irreversible Ki (MLAIR)** model. The 2-tissue compartment model structure with  $k_4=0$  is assumed. A blood volume fraction  $v_b$  is taken into account, but applied to the metabolite-corrected input curve  $C_p(t)$  rather than the whole-blood activity.

The operational equation of method **MLAIR1** for the the measured tissue TAC  $C_T(t)$  is given by

$$C_{Model}(t) = P_1 C_P(t) + P_2 \int_0^t C_P(\tau) d\tau + P_3 \int_0^t C_T(\tau) d\tau + P_4 \int_0^t \int_0^\tau C_P(s) ds d\tau$$

The linear coefficients are related to the model parameters as follows, allowing to calculate the influx  $K_i$  from  $P_3$  and  $P_4$ .

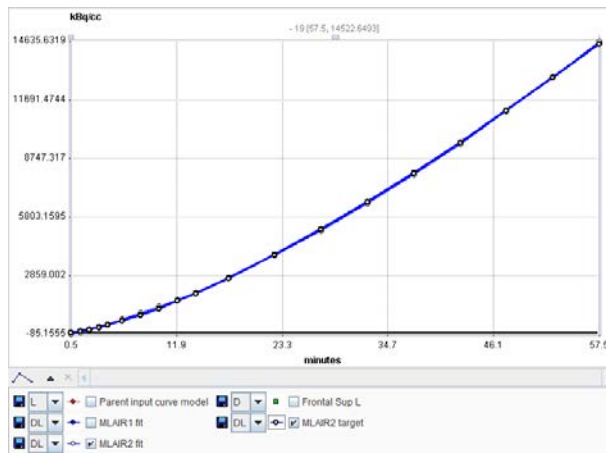
$$\begin{aligned} P_1 &= v_B & P_3 &= -(k_2 + k_3) \\ P_2 &= K_1 + k_2 v_B + k_3 v_B & P_4 &= K_1 k_3 \\ K_i &= -P_4 / P_3 \end{aligned}$$



In order to avoid the  $P_4/P_3$  division of MLAIR1 which might introduce artefacts for noisy data, a second second multilinear operational equation MLAIR2 was developed which estimates  $K_i$  as a direct regression parameter. **MLAIR2** has the following operational equation for the integrated tissue activity curve:

$$C_{Model}(t) = \int_0^t C_T(\tau) d\tau = P_1 C_P(t) + P_2 \int_0^t C_P(\tau) d\tau + P_3 C_T(t) + P_4 \int_0^t \int_0^\tau C_P(s) ds d\tau$$

$$\begin{aligned} P_1 &= v_B / (k_2 + k_3) & P_3 &= -1 / (k_2 + k_3) \\ P_2 &= K_1 / (k_2 + k_3) + v_B & P_4 &= K_i \end{aligned}$$



## Parameter Fitting

As soon as the **Irreversible Ki (MLAIR)** model is selected it fits the MLAIR1 and MLAIR2 model curves to the data using singular value decomposition and shows the regression coefficients. The parameter of interest is  $K_i$  which is calculated with both methods: **Ki(MLAIR1)** and **Ki(MLAIR2)**.

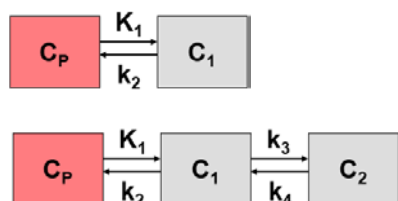
The curve area initially shows the tissue TAC together with the **MLAIR1** fit, whereas the MLAIR2-related curves are hidden because they have a completely different dynamic range. However, they can easily be shown by enabling the respective check boxes in the curve control area, as illustrated above.

## Reference:

1. Kim SJ, Lee JS, Kim YK, Frost J, Wand G, McCaul ME, Lee DS: Multiple linear analysis methods for the quantification of irreversibly binding radiotracers. *J Cereb Blood Flow Metab* 2008, 28(12):1965-1977. DOI <http://dx.doi.org/10.1038/jcbfm.2008.84>

## Logan Plot

The Logan plot has been developed by Logan et al. [1] for ligands that bind reversibly to receptors and enzymes and is used for estimating the total distribution volume  $V_T$ . Its results can be interpreted with respect to the 1- and 2-tissue compartment models.

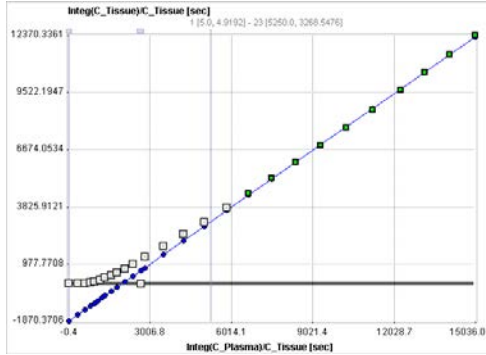


## Operational Model Curve

The Logan plot belongs to a group of *Graphical Analysis* techniques, whereby the measured tissue TAC  $C_T(t)$  undergoes a mathematical transformation and is plotted against some sort of "normalized time". The Logan plot is given by the expression

$$\frac{\int_0^t C_T(\tau) d\tau}{C_T(t)} = K \frac{\int_0^t C_P(\tau) d\tau}{C_T(t)} + b$$

with the input curve  $C_p(t)$ . This means that the tissue activity integrated from the time of injection is divided by the instantaneous tissue activity, and plotted at a "normalized time" (integral of the input curve from the injection time divided by the instantaneous tissue activity). For systems with reversible compartments this plot will result in a straight line after an equilibration time  $t^*$ .



In the derivation of the Logan plot the PET signal is described as a sum of tissue activity plus a fractional plasma signal

$$C_{Model}(t) = C_T(t) + v_p C_p(t)$$

unlike the operational equation of the compartment model. Under these premises the slope represents the total distribution volume  $V_T$  plus the plasma space  $v_p$  in the VOI, which is usually neglected. Therefore

$$K \cong V_T = K_1 / k_2 \quad \text{1 - tissue compartment model}$$

$$K \cong V_T = K_1 / k_2 (1 + k_3 / k_4) \quad \text{2 - tissue compartment model}$$

It has been found that the Logan plot is susceptible to noise in the data. Noise causes the true  $V_T$  to be underestimated, to a degree which not only depends on the noise level, but also on the local kinetics. The underestimation problem is particularly relevant in parametric mapping, where the pixelwise TACs suffer from a high noise level.

The reason for the underestimation effect is the fact that noise is not only present in the y-values (dependent variable) as the linear regression assumes, but also in the x-values (independent variable). To arrive at more accurate results it was therefore proposed to measure the residuals perpendicular to the regression line, rather than vertical to the x-axis [2].

## Parameter Fitting

The **Logan Plot** model calculates and displays the measurements transformed as described by the formula above. It allows to fit a regression line within a range defined by the start time of the linear section  $t^*$ . The results are the distribution volume (slope) and the intercept. There is also an error criterion **Max Err** to fit  $t^*$ . For instance, if **Max Err** is set to 10% and the fit box of  $t^*$  is checked, the model searches the earliest sample so that the deviation between the regression and all measurements is less than 10%. Samples earlier than the  $t^*$  time are disregarded for regression and thus painted in gray. Note that  $t^*$  must be specified in real acquisition time, although the x-axis units are in "normalized time". The corresponding



normalized time which can be looked up in the plot is shown as a non-fitable result parameter **Start**.

The regression line is calculated using the traditional and the perpendicular distances, resulting in the total distribution volumes **Vt** and **Vt\_perpend**, respectively. It has been found that **Vt** has a tendency to underestimate the distribution volume at increasing noise levels. **Vt\_perpend** shows less bias due to noise, but suffers from a somewhat increased variability.

## References

1. Logan J, Fowler JS, Volkow ND, Wolf AP, Dewey SL, Schlyer DJ, MacGregor RR, Hitzemann R, Bendriem B, Gatley SJ et al: Graphical analysis of reversible radioligand binding from time-activity measurements applied to [N-11C-methyl]-(-)-cocaine PET studies in human subjects. J Cereb Blood Flow Metab 1990, 10(5):740-747. DOI <http://dx.doi.org/10.1038/jcbfm.1990.127>
2. Varga J, Szabo Z: Modified regression model for the Logan plot. J Cereb Blood Flow Metab 2002, 22(2):240-244. DOI <http://dx.doi.org/10.1097/00004647-200202000-00012>

## RE-GP Analysis

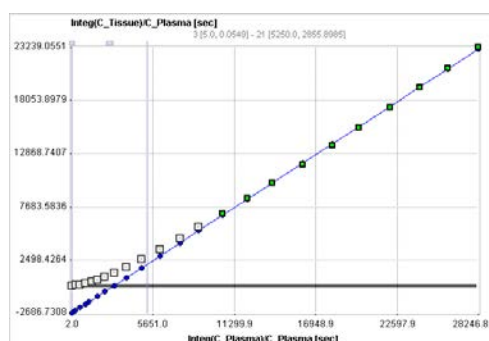
In 2009 Zhou et al. introduced a new graphical method, the Relative-Equilibrium (RE) plot [1]. It can be applied with a plasma input curve for the calculation of the distribution volume, and with a reference tissue curve for the calculation of the binding potential. It was shown with Raclopride data and with simulations, that unlike the Logan plot the RE plot is not suffering from bias due to high noise levels. As a consequence, the results obtained with VOI-averaged TACs is consistent to the results obtained in pixel-wise applications.

However, it was found that violation of the relative equilibrium condition did introduce bias. To compensate this bias Zhou et al [2] combined the RE plot with the Patlak plot in a bi-graphical manner called the **RE-GP Analysis**.

### Operational Model Curve

The operational equation of the RE Plot is given by:

$$\frac{\int_0^t C_T(\tau) d\tau}{C_p(t)} = V_{TRE} \frac{\int_0^t C_p(\tau) d\tau}{C_p(t)} + \alpha$$



For the RE plot to be applicable there must exist a time  $t^*$  after which two conditions are fulfilled:

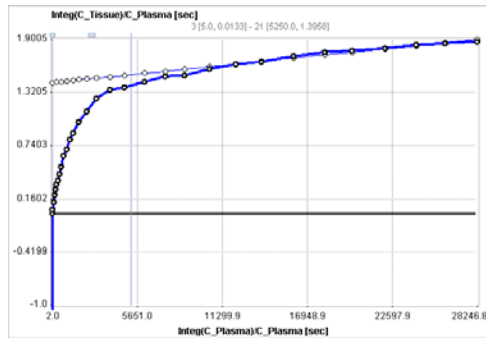
- 1) The plasma input curve must be mono-exponential. This condition can be verified by fitting a single exponential to the late part of the plasma curve on the **Blood** tab of PKIN.

The ratio of  $C_T/C_p$  is constant. This condition can be verified by switching the **KM** model to the **Tissue/Plasma Ratio** model.

Under these conditions the tracer in all tissue compartments reaches equilibrium relative to plasma. Note that the conditions must be verified explicitly, because the linear appearance of the RE plot is not a sufficient criterion.

Violation of the relative equilibrium condition above introduces bias. To compensate this bias the RE plot was combined with the Patlak plot

$$\frac{C_T(t)}{C_p(t)} = K \frac{\int_0^t C_p(\tau) d\tau}{C_p(t)} + \beta$$



using the same  $t^*$  for fitting two respective regression lines. A consistent and unbiased distribution volume is then obtained by combining the slopes and intercepts of the two plots:

$$V_T = V_{TRE} - \frac{\alpha K}{\beta}$$

For the pixel-wise application of the RE-GP Analysis the results of the Patlak plot are smoothed, so that the calculation turns into

$$V_T = V_{TRE} - \frac{\alpha K_s}{\beta_s}$$

where  $K_s$  and  $\beta_s$  are obtained from spatially smoothed maps of  $K$  and  $\beta$ .

## Parameter Fitting

The **RE-GP Analysis** model calculates and displays the measurements transformed as described by the RE plot formula above. It allows fitting a regression line using the data points after a time  $t^*$ . Note that  $t^*$  must be specified in real acquisition time, although the x-axis units are in "normalized time". The corresponding normalized time which can be looked up in the plot is shown as a non-fitable result parameter **Start**.

If  $t^*$  is changed to define a new data segment, the program finds the closest acquisition start time, fits the two regression lines to the RE plot and the Patlak plot, and updates the calculated parameters. The main outcome is the distribution volumes calculated with the RE-GP analysis (**Vt-REGP**). For a comparison, Vt calculated with the RE plot alone (**Vt-RE**) is also shown.

Per default, only the curves of the RE plot and its regression line are shown in the curve area. However, the Patlak plot and its regression line can also be visualized by enabling their boxes in the curve control area. Note that the values of the Patlak plot may have a very different dynamic range than the RE plot. Therefore it is recommended switching off the RE curves when switching on the Patlak ones.

## References

1. Zhou Y, Ye W, Brasic JR, Crabb AH, Hilton J, Wong DF: A consistent and efficient graphical analysis method to improve the quantification of reversible tracer binding in radioligand receptor dynamic PET studies. *Neuroimage* 2009, 44(3):661-670. DOI <http://dx.doi.org/10.1016/j.neuroimage.2008.09.021>
2. Zhou Y, Ye W, Brasic JR, Wong DF: Multi-graphical analysis of dynamic PET. *Neuroimage* 2010, 49(4):2947-2957. DOI <http://dx.doi.org/10.1016/j.neuroimage.2009.11.028>

## Ito Plot

In 2010 Ito et al. introduced a new graphical method for the analysis of reversible receptor ligand binding [1]. It is applied with a plasma input curve for the calculation of the total distribution volume  $V_T$ , the distribution volume of the nondisplaceable compartment  $V_{ND}$ , and therefore also allows calculation of the binding potential  $BP_{ND}$ . Furthermore, the shape of the plot gives an indication whether there the tissue includes is specific binding or not not.

### Operational Model Curve

The equation of the graphical plot called is given by the equation

$$\frac{C_T}{\int_0^t C_P(\tau) d\tau} = a - b \frac{\int_0^t C_T(\tau) d\tau}{\int_0^t C_P(\tau) d\tau}$$

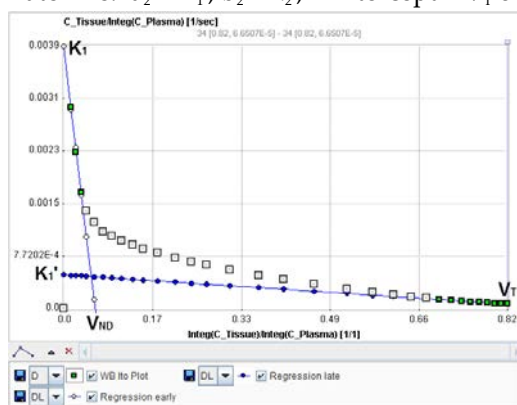
The following cases can be distinguished:

- 1) The plot is a straight line, indicating the *absence* of specific binding. In this case the line equation parameters have the following interpretation in terms of a 2-tissue compartment model:  
 $a = K_1$  (y-intercept),  $b = k_2$  (slope), and the x-intercept equals  $V_{ND}$ .

The plot has a curved shape as illustrated below, indicating the presence of specific binding. In this case two regression lines are fitted to the plot: a line to the early part within a short time segment  $T_1$  to  $T_2$ , and a line to the end part after an equilibration time  $t^*$ . The parameters of the resulting lines can be interpreted as follows:

Early line:  $a_1 = K_1$ ,  $b_1 = k_2$ , x-intercept =  $V_{ND}$  of a 2-tissue compartment model.

Late line:  $a_2 = K_1'$ ,  $b_2 = k_2'$ , x-intercept =  $V_T$  of a 1-tissue compartment model.



The binding potential can finally be calculated by  $BP_{ND} = (V_T - V_{ND})/V_{ND}$

## Parameter Fitting

The **Ito Plot** model calculates and displays the measurements transformed as described by the Ito plot formula above. It allows fitting two regression lines as follows:

- 1) The first regression line is fitted within the data segment specified by the times **T1** and **T2** (in acquisition time).

The second regression line is fitted to the late data segment starting at equilibration time **t\***.

If any of the times is changed to define a new data segment, the program finds the closest acquisition start time, fits the two regression lines, and updates the calculated parameters.

**t\*** can be specified manually, or a value estimated using an error criterion **Max Err**. For instance, if **Max Err** is set to 10% and the fit box of **t\*** is checked, the model searches the earliest sample so that the deviation between the regression line and all measurements is less than 10%. Samples earlier than the **t\*** time are disregarded for regression and thus painted in gray. Note that **t\*** must be specified in real acquisition time, although the x-axis units are in "normalized time". The corresponding normalized time which can be looked up in the plot is shown as a non-fitable result parameter **Start**. In order to apply the analysis to the same data segment in all regions, please switch off the fit box of **t\***, propagate the model with the **Copy to all Regions** button, and then activate **Fit all regions**.

Per default, only the Ito Plot and the late regression line is shown in the curve area.

However, the early regression line can also be visualized as illustrated above by enabling its box in the curve control area.

## References

1. Ito H, Yokoi T, Ikoma Y, Shidahara M, Seki C, Naganawa M, Takahashi H, Takano H, Kimura Y, Ichise M et al: A new graphic plot analysis for determination of neuroreceptor binding in positron emission tomography studies. *Neuroimage* 2009, 49(1):578-586. DOI <http://dx.doi.org/10.1016/j.neuroimage.2009.07.021>

## Ichise Multilinear Analysis MA1

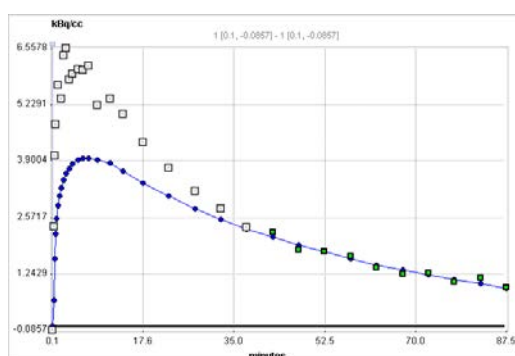
Ichise's MA1 analysis method [1] is a technique for calculating the total distribution volume for receptor tracers with reversible binding. It is a further development of the *Logan Plot* (on page 166) aimed at minimizing the bias induced by noise in the measurements.

### Operational Model Curve

The following bilinear relationship was derived

$$C_{Model}(t) = -\frac{V_T}{b} \int_0^t C_P(\tau) d\tau + \frac{1}{b} \int_0^t C_T(\tau) d\tau$$

where  $C_T(t)$  represents the tissue time-activity curve,  $C_P(t)$  the plasma activity,  $V_T$  the total distribution volume, and  $b$  the intercept of the Logan plot which becomes constant after an equilibration time  $t^*$ .



Based on simulation and experimental data the authors show that MA1 demonstrates the largest bias reduction among several methods. Therefore they conclude, that MA1 is the method of choice for calculating the total distribution volume, if  $t^*$  can accurately be defined.

### Parameter Fitting

The bilinear regression is performed using singular value decomposition. The result are the two coefficients  $V_T/b$  and  $b$ , from which  $Vt$  can easily be derived.

The MA1 method requires specification of the equilibration time as the  $t^*$  parameter. As it equals the equilibration time of the Logan plot, it is possible to perform a *Logan plot* (on page 166) first and use the resulting Logan  $t^*$ .

$t^*$  can also be estimated using an error criterion **Max Err.** For instance, if **Max Err.** is set to 10% and the fit box of  $t^*$  is checked, the model searches the earliest sample so that the deviation between the regression and all measurements is less than 10%. Samples earlier than the  $t^*$  time are disregarded for regression and thus painted in gray. In order to apply the analysis to the same data segment in all regions, please switch off the fit box of  $t^*$ , propagate the model with the **Copy to all Regions** button, and then activate **Fit all regions**.

## Reference

1. Ichise M, Toyama H, Innis RB, Carson RE: Strategies to improve neuroreceptor parameter estimation by linear regression analysis. *J Cereb Blood Flow Metab* 2002, 22(10):1271-1281. DOI <http://dx.doi.org/10.1097/00004647-200210000-00015>

## Ichise Multilinear Analysis MA2

Ichise's MA2 analysis method [1] is another alternative technique developed to calculate the total distribution volume of reversible receptor systems with minimal bias. It is particularly suitable for tracers with slow kinetics and low to moderate noise.

The MA2 method has two advantages:

- 1) It is independent of an equilibration time, so that the data from the first acquisition can be included into the regression.

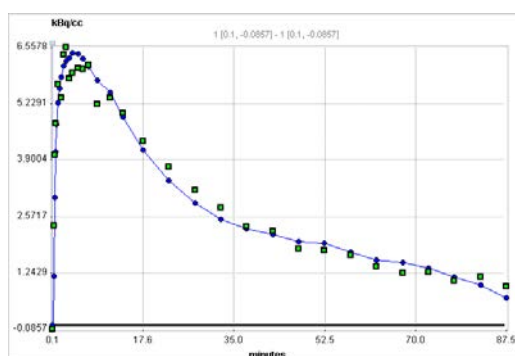
An estimate of the specific distribution volume is also obtained. The authors state, that although the method has been derived with the 2-tissue model, it still shows a good performance with the data representing only 1-tissue characteristics.

## Operational Model Curve

Based on the 2-tissue compartment model equations the following multi-linear relationship was derived [34]:

$$C_{Model}(t) = p_1 \int_0^t \int_0^s C_p(\tau) d\tau ds + p_2 \int_0^t \int_0^s C_T(\tau) d\tau ds + p_3 \int_0^t C_T(\tau) d\tau + p_4 \int_0^t C_p(\tau) d\tau$$

where  $C_T(t)$  represents the tissue time-activity curve, and  $C_p(t)$  the plasma activity.



A multi-linear regression can be performed to calculate the four regression coefficients from the transformed data. The total distribution volume  $V_T$  is then calculated as the ratio of the first two regression coefficients, and the distribution volume of specific binding  $V_s$  by the expressions

$$V_T = -\frac{p_1}{p_2} \quad V_s = \frac{-p_1(p_1 + p_3p_4) + p_2p_4^2}{p_2(p_1 + p_3p_4)}$$

## Parameter Fitting

The multi-linear regression is performed using a singular value decomposition, resulting in the 4 regression parameters **p1**, **p2**, **p3**, **p4**, and the derived distribution volumes **Vt** and **Vs**. Although no equilibration time is required for MA2, there is a **t\*** parameter to disregard early samples from the regression as for the *Logan plot* (on page 166) and *MA1* (on page 172).

## Reference

1. Ichise M, Toyama H, Innis RB, Carson RE: Strategies to improve neuroreceptor parameter estimation by linear regression analysis. J Cereb Blood Flow Metab 2002, 22(10):1271-1281. DOI <http://dx.doi.org/10.1097/00004647-200210000-00015>

## Bolus/Infusion Optimization

When a compartment system has equilibrated, the total distribution volume can be calculated easily as the ratio of tracer concentration in tissue to the metabolite-corrected plasma concentration with a single static scan. It has been found that the time required to reach equilibrium can be shortened by an optimized tracer delivery. A setup which does not require a sophisticated tracer delivery system is to apply an initial bolus and continue with a constant infusion.

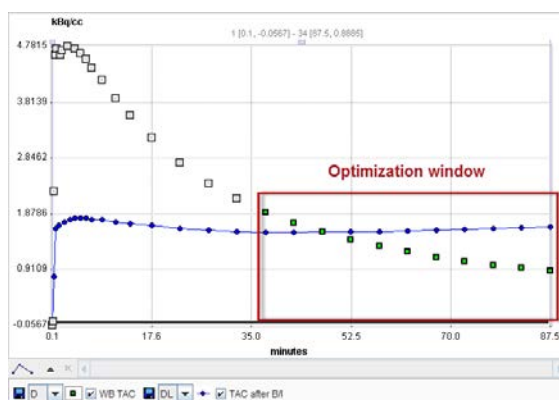
Carson et al. [1] have developed a method to optimize the ratio between the activity given as the initial bolus and the activity level of the subsequent infusion for quickly reaching an equilibrium (Appendix B in [1]). It is assumed that tissue TACs as well as the input curve have been measured beforehand in a bolus experiment. Ideally then, the measured activities represent the impulse response of the system. Under this assumption it is possible to calculate the tissue response for any given input curve by just convolving the TACs with the input curve.

## Operational Model Curve

For a bolus and infusion (B/I) experiment the input function can be described as an initial impulse followed by a step function, and the convolution can be analytically solved. The resulting model curve

$$C_{\text{Model}}(t) = \frac{K_{\text{bol}} C_T(t) + \int_0^t C_T(\tau) d\tau}{K_{\text{bol}} + T}$$

is a function of the bolus fraction and the infusion duration.  $K_{bol}$  [min] defines the dose of the bolus expressed as minutes of infusion, and  $T$  is the total duration of the infusion (end of last frame).



## Parameter Fitting

It is assumed that the data of a bolus study has been loaded. When the **Bolus/Infusion Opt** model is selected, the expected tissue activity according to the formula above is calculated using the default parameters, and shown as the model curve.

**Kbol** can be estimated. The user has to enter a reasonable equilibration time as the input parameter **Start Equilibr**. When **Fit current region** is activated, the optimal **Kbol** is calculated. The criterion for an equilibrium is that the calculated activities from **Start Equilibr** to the end of the scan are as constant as possible. Also, please note the following:

- 1) The same formulation holds for both TACs and input curves. However,  $K_{bol}$  can only be fitted with the TAC curve, not the input curve. For the blood curve, the **Bolus/Infusion Opt** model is only usable for visualizing the plasma response with an entered  $K_{bol}$  value which was obtained in TAC fitting.

If the **Bolus/Infusion Opt** model is applied to a TAC and the **Fit blood** is enabled in the **Extras** panel, the **Bolus/Infusion Opt** model is also applied to the plasma curve during the fit using the plasma model curve. Therefore the *plasma model should be an interpolation model, NOT THE B/I model!* **Kbol** is a common parameter in the fit, and the criterion is that both the expected TAC and the expected input curve are constant.

The **Bolus/Infusion Opt** model can be used for coupled fitting to find the optimal dose for several tissues. In a coupled fit, only the TACs are used, and the input curve is currently neglected (as opposed to 2).

The residual weighting of all curves involved in the fit should be set to constant weighting.

## Reference

1. Carson RE, Channing MA, Blasberg RG, Dunn BB, Cohen RM, Rice KC, Herscovitch P: Comparison of bolus and infusion methods for receptor quantitation: application to [18F]cyclofoxy and positron emission tomography. J Cereb Blood Flow Metab 1993, 13(1):24-42. DOI <http://dx.doi.org/10.1038/jcbfm.1993.6>



## New Topic (8)

The **Residence Times** model performs pre-processing steps for the calculation of the absorbed dose from diagnostic or therapeutic radiopharmaceuticals. Given the time-course of the activity in a volume of tissue, it calculates the *Residence Time*, which should better be called the *Normalized Cumulated Activity* [1] to avoid confusion of its meaning. It represents the total number of disintegrations which have occurred during an integration time per unit administered activity. Ideally, integration is performed from the time of administration to infinity. However, as it is widely used in literature, the notion Residence Time will be used in the remainder of this section.

The organ residence times resulting from the **Residence Times** model can serve as input to a program such as OLINDA [2] for the actual calculation of the absorbed organ doses.

### Operational Equations

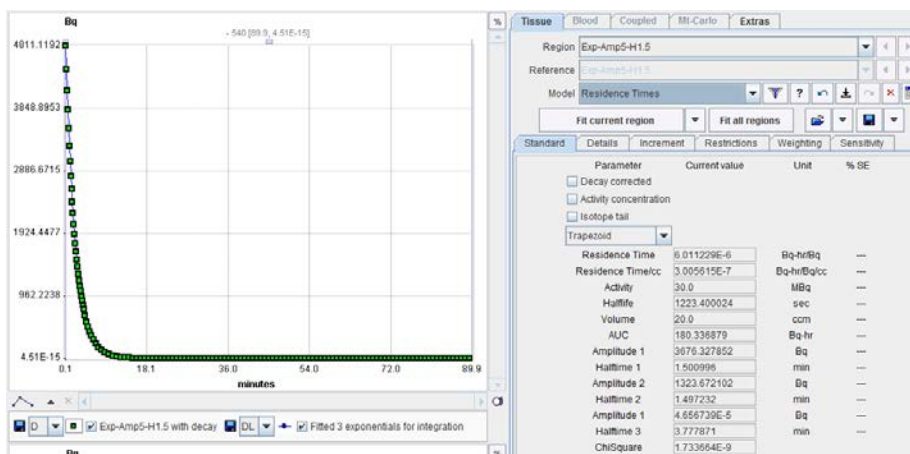
Given an activity  $A_0$  applied to a subject, and a measured (not decay corrected) activity  $A(t)$  in an organ, the residence time  $\tau$  is calculated by

$$\tau = \frac{\int_0^{\infty} A(t) dt}{A_0}$$

Note that the unit of  $\tau$  is usually given as [Bq·hr/Bq]. The measured part of the organ activity can easily be numerically integrated by the trapezoidal rule. For the unknown remainder of  $A(t)$  till infinity it is a conservative assumption to apply the radioactive decay of the isotope. This exponential area can easily be calculated and added to the trapezoidal area. Alternatively, if the measured activity curve has a dominant washout shape, a sum of exponentials can be fitted to the measured data and the entire integral algebraically calculated.

The **Residence Times** model in PKIN supports both the trapezoidal integration approach as well as the use of fitted exponentials. It furthermore supports some paracticalities such as

- ▶ conversion of an average activity concentration in [kBq/cc] to activity in [Bq] by multiplication with the VOI volume;
- ▶ reversion of decay correction;



## Model Input Parameters

The Residence Times model has 4 input parameters which need to be specified interactively by the user. These settings can easily be propagated from one region to all others with the **Model & Par** button.

**Decay corrected** Check this box if the loaded curves have been decay corrected as is usually the case for PET imaging studies. The **Half-life** of the radioisotope will be used to undo the decay correction.

**Activity concentration** Check this box if the loaded curves represent activity concentration rather than the total activity in a region. The **volume** of the region will be multiplied with the signal to calculate activity in [Bq].

**Isotope tail** Check this box to use the radioactive decay shape for the integration from the end of the last frame to infinity. This option is only relevant if exponentials are fitted to the measurement. For the **Rectangle** and **Trapezoid** methods it will always be added.

**Integration** The integration method can be selected in the option list: **Rectangle**, **Trapezoid**, **1 Exponential**, **2 Exponentials**, **3 Exponentials**.

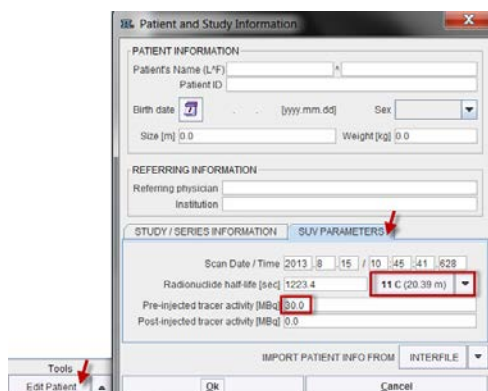
The **Rectangle** method is the natural selection for PET values which represent the time average during the frame duration. If there are gaps between the PET frames, the uncovered area is approximated by the trapezoidal rule.

The **Trapezoid** method uses trapezoidal areas between the frame mid-times.

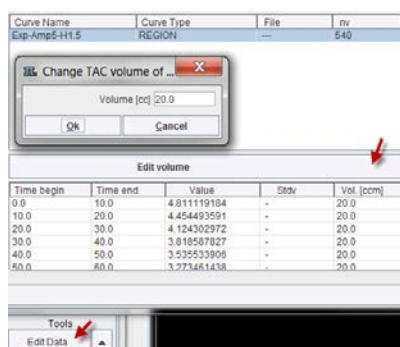
With an **Exponential** selection, the specified number of decaying exponentials is fitted to the measurements, and used for analytical integration. Note that samples marked as invalid are neglected in the fit.

## Editing of Constituents

Depending on the import of the activity curves, isotope and activity information may or may not be available. They can be edited in the **Patient and Study Information** window as illustrated below. Activate **Edit Patient** in the **Tools** list, select SUV PARAMETERS, and enter **Radionuclide half-life** and **Pre-injected tracer activity**. Note that the activity needs to be calibrated to time 0 of the activity curve.



The region volume is available if the TAC was generated in PMOD's View tool and transferred to PKIN. Otherwise the volume has to be entered as illustrated below. Activate **Edit Data** in the **Tools** list, select a region in the upper list, then activate **Edit volume** from the curve tools and enter the **Volume** number.



## Model Output Parameters

The Residence Times model has 4 input parameters which need to be specified interactively by the user. These settings can easily be propagated from one region to all others with the **Model & Par** button.

As soon as the **Fit Region** button is activated, the fits and calculations are performed, resulting in a list of parameters:

**Residence Time [Bq·hr/Bq]** Total number of disintegrations in region from time 0 until infinity. The main result and equal to **AUC/Activity**.

<b>Residence Time/cc</b> <b>[Bq/cc·hr/Bq]</b>	Total number of disintegrations per unit volume in the region. Equal to <b>AUC/(Activity·Volume)</b> .  If the organ has deliberately been outlined within the organ to reduce the partial-volume problem, <b>Residence Time/cc</b> may be multiplied with the actual organ size to approximate the activity taken up.  The residence time for standard phantom organs can be easily be calculated by multiplying the tabulated organ mass with <b>Residence Time/cc</b> and dividing by the organ density (usually 1g/ccm).
<b>Activity [MBq]</b>	Administered activity. Shown for verification only.
<b>Halflife [sec]</b>	Isotope halflife. Shown for verification only.
<b>Volume [ccm]</b>	Region volume. Shown for verification only.
<b>AUC [Bq·hr]</b>	Integral of the activity curve from time 0 till infinity.
<b>Amplitude 1, 2, 3 [Bq]</b> <b>Halftime 1, 2, 3 [min]</b>	Amplitudes and halftimes of the fitted exponentials. In the case of 1 exponential a linear regression is applied, whereas iterative fitting is used with 2 or 3 exponentials.

## Reference

1. Stabin MG: Fundamentals of Nuclear Medicine Dosimetry. Springer; 2008. DOI <http://dx.doi.org/10.1007/978-0-387-74579-4>
2. Stabin MG, Sparks RB, Crowe E: OLINDA/EXM: the second-generation personal computer software for internal dose assessment in nuclear medicine. J Nucl Med 2005, 46(6):1023-1027.

## Area under Curve (AUC)

This is an auxiliary model which cannot be used for fitting. It calculates the following curves from the input curve  $C_p(t)$  and the selected regional tissue TAC  $C_T(t)$  at the frame mid-times:

- 1) The AUC of the tissue TAC as a function of time:

$$AUC_T(t) = \int_0^t C_T(\tau) d\tau$$

The AUC of the input curve as a function of time:

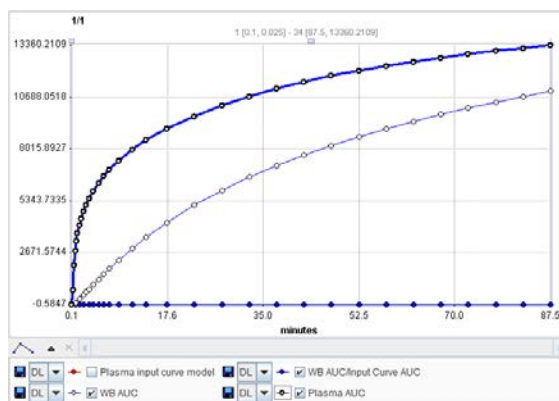
$$AUC_P(t) = \int_0^t C_P(\tau) d\tau$$

Note that the integration uses the interpolation model selected for the blood curves involved in the calculation of the input curve.

The ratio of the two AUCs as a function of time

$$AUC_{ratio}(t) = \int_0^t C_T(\tau) d\tau / \int_0^t C_P(\tau) d\tau$$

It provides an indication of the tissue accumulation of the tracer [1].



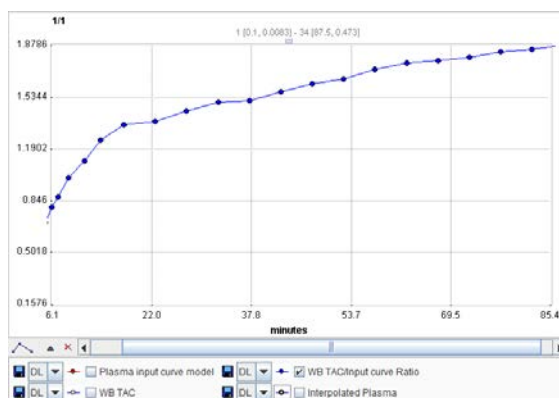
As the ratio curve has a completely different scaling than the curve integrals, it appears along the zero line. It can be shown with appropriate scaling by hiding all other curves. The easiest way to export the values is using the **Save all curves** or the **View values** option in the context menu (right click into curve area).

## Reference

1. Eyal S, Chung FS, Muzi M, Link JM, Mankoff DA, Kaddoumi A, O'Sullivan F, Hebert MF, Unadkat JD: Simultaneous PET Imaging of P-Glycoprotein Inhibition in Multiple Tissues in the Pregnant Nonhuman Primate. *Journal of Nuclear Medicine* 2009, 50(5):798-806. **DOI** <http://dx.doi.org/10.2967/jnumed.108.059360>

## Tissue/Plasma Ratio

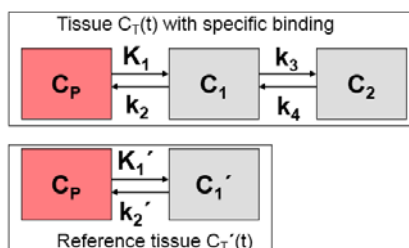
This auxiliary model simply divides the tissue TAC at each frame mid-time by the interpolated plasma activity at the same time and displays it as a curve called **Tissue/Input curve Ratio**.



Under equilibrium conditions this ratio is equal to the total distribution volume of tracer in tissue. Therefore, the plot of the **Tissue/Input curve Ratio** against time is a quick means to check whether the system has equilibrated (plot becomes constant) or not.

## Tissue Ratio Methods

This auxiliary model is aimed at receptor tracers for which a reference tissue without specific binding is available. It allows getting a quick estimate of the binding potential  $BP_{ND}$  using two different methods as described by Ito et al. [1].



At the time, when specific binding  $C_2$  peaks, its derivative equals zero (transient equilibrium) and  $BP_{ND} = k_3/k_4 = C_2/C_1$  (non-specific/specific binding).

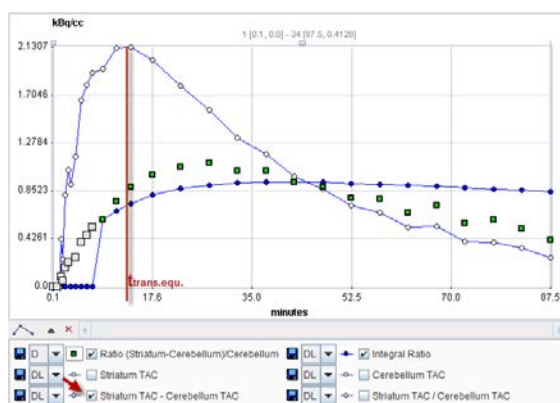
In practice, however, neither the  $C_1$  part nor the  $C_2$  part of a measured tissue TAC are known. As a pragmatic solution it is assumed that the non-displaceable compartment is equal among tissues, so the  $C_T'(t)$  of reference tissue without specific binding is regarded as  $C_{1'}$ , and  $C_2$  is calculated by  $C_T(t) - C_T'(t)$ .

## Operational Model Curve

Two different ratios are calculated by the **Tissue Ratio Methods** model:

$$BP_{ND} = \frac{k_3}{k_4} = \frac{C_2(t_{trans.equ.})}{C_1(t_{trans.equ.})} = \frac{C_T(t_{trans.equ.}) - C_T'(t_{trans.equ.})}{C_T'(t_{trans.equ.})}$$

This ratio is calculated for all times, but it is *only valid at the time of transient equilibrium*. There is no automatic routine for detecting this peak. Rather, the user has to find this time by looking at the difference curve (enable check box).



The second ratio calculated is called the *interval method* in [1].

$$BP_{ND} = \frac{k_3}{k_4} = \frac{\int_{t_b}^{t_e} C_2(t) dt}{\int_{t_b}^{t_e} C_1(t) dt}$$

Here, the two curves are integrated in a time interval  $[t_b, t_e]$  which must include the time of transient equilibrium ( $t_b=9\text{min}$ .  $t_e=45\text{ min}$  for raclopride, where the time of transient equilibrium is in the range 20 to 24 min [1]).

## Implementation

- 1) After switching to **Tissue Ratio Methods** a suitable reference tissue without specific binding must be selected.

The integration start time of the interval method must be specified by the user as the input parameter **Integral start time**.

The ratios are calculated for all times (for the integral method only after the **Integral start time**) and presented as curves. These curves can easily be exported as values using the *save button* (on page 46).

## Reference

1. Ito H, Hietala J, Blomqvist G, Halldin C, Farde L: Comparison of the transient equilibrium and continuous infusion method for quantitative PET analysis of [11C]raclopride binding. J Cereb Blood Flow Metab 1998, 18(9):941-950. DOI <http://dx.doi.org/10.1097/00004647-199809000-00003>

## Tracer Retention Fraction

The *Retention Fraction* is the fraction of the total tracer delivered to an organ that is extracted into and retained by the tissue. It is the residual after clearances of the vascular component and the portion of the tracer that rapidly back diffuses from tissue to blood. Usually this fraction of tracer is sequestered in more slowly turning over metabolic or membrane binding processes. [1]

## Operational Equation

Given a tissue TAC  $C_T(t)$  and the input curve  $C_p(t)$  this auxiliary model allows calculating the retention fraction  $R$  by [2]:

$$R = \frac{\int_{t_b}^{t_e} C_T(\tau) d\tau}{\int_0^{t_e} C_p(\tau) d\tau}$$

The denominator with the input curve integrated from the time of injection represents the available tracer, the numerator with the integral of the late tissue uptake the actually extracted tracer.

## Implementation

The **Tracer Retention** auxiliary model has two input parameters **Start** and **End** for the specification of  $t_s$  and  $t_e$  in the integration formula above, respectively. The curve display shows the input curve interpolated at the frame mid-times.

## References

1. see Glossary in Phelps ME: PET : molecular imaging and its biological applications. New York: Springer; 2004.
2. Hutchins GD, Chen T, Carlson KA, Fain RL, Winkle W, Vavrek T, Mock BH, Zipes DP: PET imaging of oxidative metabolism abnormalities in sympathetically denervated canine myocardium. J Nucl Med 1999, 40(5):846-853.

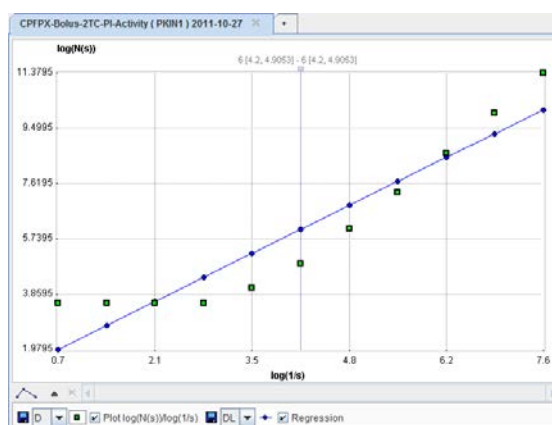
## Fractal Dimension

The **Fractal Dimension** model measures the complexity of a 2-dimensional structure by calculating its box-counting dimension [1]. This concept has been applied in oncologic studies for assessing the heterogeneity of tissue kinetics [2].

## Operational Equation

The idea is to subdivide the area under the tissue TAC into a number of square boxes and simply count the number of boxes containing some part of the structure. The mesh size is defined as  $s$ , so  $1/s$  gives the number of segments in each of the 2 dimensions. For instance, specifying  $1/s=5$  therefore means a subdivision into  $5*5=25$  boxes.

The counting process is performed with increasing number of intervals up to the specified  $1/s$ . Next, the data are plotted in a double-logarithmic way, namely  $\log(N(s))$  on the y axis and  $\log(1/s)$  on the x-axis.



The box-counting dimension is finally obtained as the slope of a linear regression through the plotted points.



## Implementation

After switching to the **Fractal dimension** model, two input parameters are available for specifying the box-counting process: **1/s**, and **Maximal value** the highest TAC value which might occur in the data.

## References

1. Peitgen H-O, Jürgens H, Saupe D: Chaos and fractals : new frontiers of science. New York: Springer-Verlag; 1992.
2. Strauss LG, Dimitrakopoulou-Strauss A, Koczan D, Bernd L, Haberkorn U, Ewerbeck V, Thiesen HJ: 18F-FDG kinetics and gene expression in giant cell tumors. J Nucl Med 2004, 45(9):1528-1535.

# Activity Models for Whole-Blood and Plasma

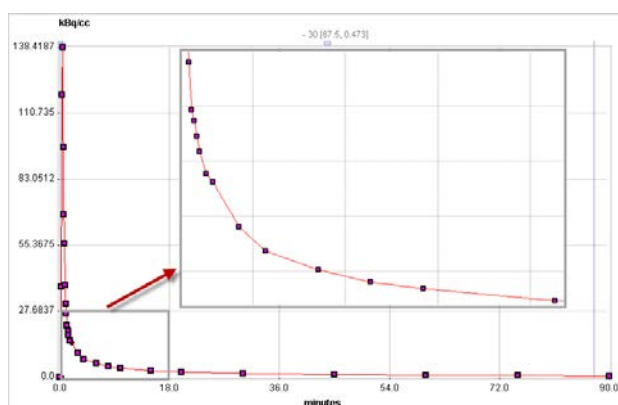
## Linear Activity Interpolation

The blood and plasma activity concentrations require interpolation procedures for kinetic modeling, as described *above* (on page 51).

### Operational Model Curve

The **Lin. Interpolation** model interpolates linearly between the measured activity values. Outside the measured values the following interpolation rules apply:

- ▶ If the first measurements occurs at  $t < 0$ , the activity is assumed to be zero for all earlier times.
- ▶ If the first measurements occurs at  $t > 0$ , the activity is assumed to be zero for  $t \leq 0$ . Activity values from  $t = 0$  up to the first measurement are calculated by linear interpolation from activity 0 at  $t = 0$ .
- ▶ The activity is assumed to remain constant after the last measurement. For short blood curves it is therefore recommended to replace the tail of the measurements by a fitted *tri-exponentia* (on page 185) model.



### Parameter Fitting

The **Lin. Interpolation** model has no fitting parameter.

The **Delay** parameter serves for correcting a timing offset between tissue and blood data. Positive delays correspond to delayed blood information and hence shift the blood curve to earlier times (to the left). The **Delay** is only relevant during the fitting of the tissue model where it can be *fitted as an additional parameter* (on page 77) to the parameters of the kinetic model.

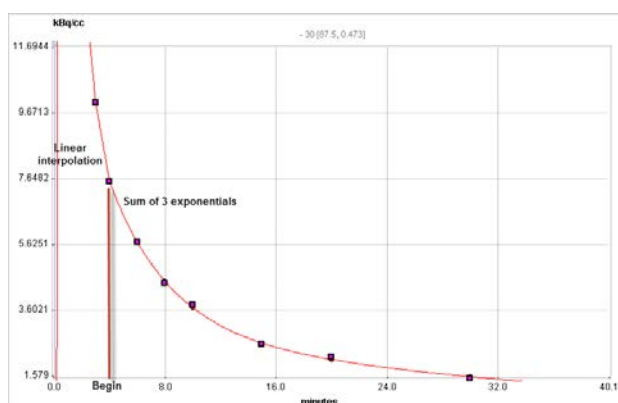
## Decaying Exponentials

The blood and plasma activity concentrations require interpolation procedures for kinetic modeling, as described *above* (on page 51).

## Operational Model Curve

The **3 Exponentials** model allows replacing the tail of the blood activity curve by a fitted sum of up to three exponentials. The **Begin** parameter defines at what time the models switches from linear to tri-exponential interpolation. The exponentials are each defined by an **Amplitude** and a **Halftime** [min] of the decay. The number of exponentials can easily be reduced by fixing one or two of the amplitudes at a value of 0.

$$f_{\text{Plasma/WB}}(t) = \begin{cases} \text{Lin.Interpol} & t < \text{Begin} \\ \sum_{i=1}^3 A_i e^{-u \ln 2 / T_i} & t \geq \text{Begin}, u = t - \text{Begin} \end{cases}$$



## Parameter Fitting

Per default only two exponentials are enabled, and the **Begin** is initialized to the time of the blood peak. Use the **Fit plasma activity** or **Fit whole blood** button to fit the two exponentials to the measurements after the specified **Begin**. If the shape cannot be described well enough, enable the third exponential by enabling the fit boxes and set **Amplitude 3** to a positive number, then **Fit** again. Possibly, **Begin** has to be set to a later time for obtaining the signal tail.

The **Delay** parameter serves for correcting a timing offset between tissue and blood data. Positive delays correspond to delayed blood information and hence shift the blood curve to earlier times (to the left). The **Delay** is only relevant during the fitting of the tissue model where it can be *fitted as an additional parameter* (on page 77) to the parameters of the kinetic model.

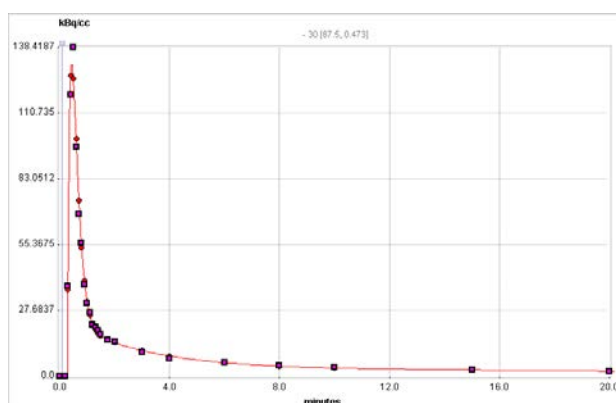
## Compartment Model with 3 Eigenvalues

The blood and plasma activity concentrations require interpolation procedures for kinetic modeling, as described *above* (on page 51).

## Operational Model Curve

The **Compartment, 3 EV** model has been developed for the FDG tracer [1]. It is the result from modeling the distribution and delivery of FDG in the circulatory system by a compartment model and results in an analytical solution with 3 eigenvalues (EV):

$$C_{continuous}(t) = \begin{cases} 0 & t < \text{Begin} \\ [A_1 u - A_2 - A_3] e^{-u \ln 2 / T_1} + A_2 e^{-u \ln 2 / T_2} + A_3 e^{-u \ln 2 / T_3} & t \geq \text{Begin}, u = t - \text{Begin} \end{cases}$$



## Parameter Fitting

Similar to the **3 Exponentials (on page 185)** model it includes a **Begin** parameter and three **Amplitudes** ( $A_1$ ,  $A_2$ ,  $A_3$ ) and a **Halftimes** ( $T_1$ ,  $T_2$ ,  $T_3$ ). However, the functional form supports a peak rising from time **Begin** followed by an exponentially decreasing tail. Use the **Fit plasma activity** or **Fit whole blood** button to fit the operational model curve to the measurements. Note that all samples are considered, not only the ones after **Begin**.

The form of the curve reacts very sensitively to changes of the parameters. The example above illustrates a peak which could not fully reach the maximum value. This model may be more suited for simulations rather than actual data processing.

The **Delay** parameter serves for correcting a timing offset between tissue and blood data. Positive delays correspond to delayed blood information and hence shift the blood curve to earlier times (to the left). The **Delay** is only relevant during the fitting of the tissue model where it can be *fitted as an additional parameter* (on page 77) to the parameters of the kinetic model.

## Reference

1. Feng D, Huang SC, Wang X: Models for computer simulation studies of input functions for tracer kinetic modeling with positron emission tomography. International journal of bio-medical computing 1993, 32(2):95-110.

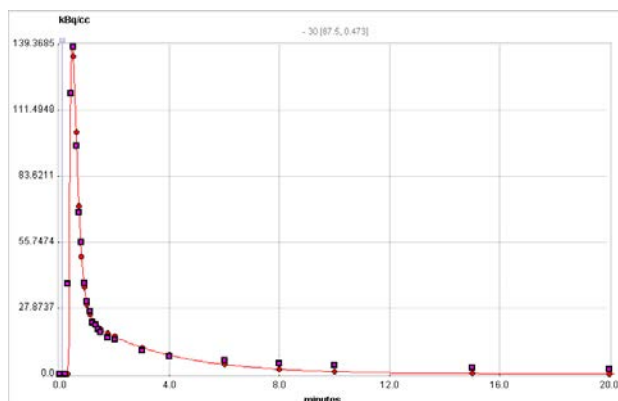
## Compartment Model with 2 Eigenvalues

The blood and plasma activity concentrations require interpolation procedures for kinetic modeling, as described *above* (on page 51).

## Operational Model Curve

The **Compart, 2 EV** model is an alternative model to the **Compart, 3 EV (on page 186)** model for the plasma activity curve of FDG [1]. It assumes repeated eigenvalues (EV) and has the following functional form:

$$C_{\text{continuous}}(t) = \begin{cases} 0 & t < \text{Begin} \\ [A_1 u - A_2] e^{-u \ln 2 / T_1} + A_2 e^{-u \ln 2 / T_2} & t \geq \text{Begin}, u = t - \text{Begin} \end{cases}$$



## Parameter Fitting

Similar to the **3 Exponentials (on page 185)** model it includes a **Begin** parameter and two **Amplitudes** ( $A_1$ ,  $A_2$ ) and a **Halftimes** ( $T_1$ ,  $T_2$ ). However, the functional form supports a peak rising from time **Begin** followed by an exponentially decreasing tail. Use the **Fit plasma activity** or **Fit whole blood** button to fit the operational model curve to the measurements. Note that all samples are considered, not only the ones after **Begin**.

The form of the curve reacts very sensitively to changes of the parameters. The example above illustrates a result is model may be more suited for simulations rather than actual data processing.

The **Delay** parameter serves for correcting a timing offset between tissue and blood data. Positive delays correspond to delayed blood information and hence shift the blood curve to earlier times (to the left). The **Delay** is only relevant during the fitting of the tissue model where it can be *fitted as an additional parameter* (on page 77) to the parameters of the kinetic model.

## Reference

1. Feng D, Huang SC, Wang X: Models for computer simulation studies of input functions for tracer kinetic modeling with positron emission tomography. International journal of biomedical computing 1993, 32(2):95-110.

## Gamma Function

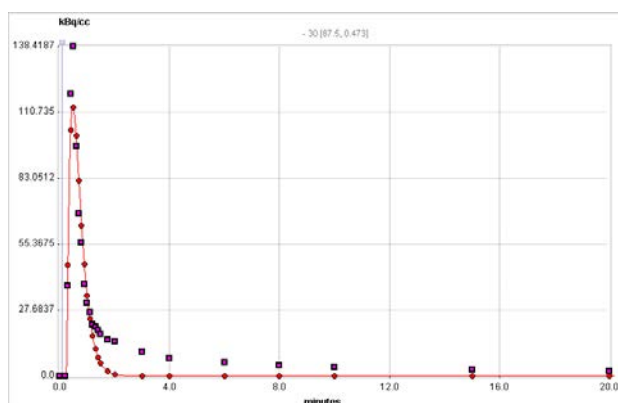
The blood and plasma activity concentrations require interpolation procedures for kinetic modeling, as described *above* (on page 51).

## Operational Model Curve

The **Gamma Function** model is defined by the following functional form:

$$C_{\text{continuous}}(t) = \begin{cases} 0 & t < \text{Begin} \\ A u^B e^{-u C} & t \geq \text{Begin}, u = t - \text{Begin} \end{cases}$$

describing a sharp peak governed followed by a quick decay towards zero.



## Parameter Fitting

The **Gamma Function** model includes a **Begin** time parameter for positioning the peak. The factor **A** scales the curve, the exponent **B** governs the rise and constant **C** the decay towards zero. Use the **Fit plasma activity** or **Fit whole blood** button to fit the operational model curve to the measurements. Note that all samples are considered, not only the ones after **Begin**.

The form of the curve reacts very sensitively to changes of the parameters. The example above illustrates the inability to fit model a peak followed by a moderate to slow decay appropriately. This model may therefore be more suited for simulations rather than actual data processing.

The **Delay** parameter serves for correcting a timing offset between tissue and blood data. Positive delays correspond to delayed blood information and hence shift the blood curve to earlier times (to the left). The **Delay** is only relevant during the fitting of the tissue model where it can be *fitted as an additional parameter* (on page 77) to the parameters of the kinetic model.

## Gamma Function plus two Decaying Exponentials

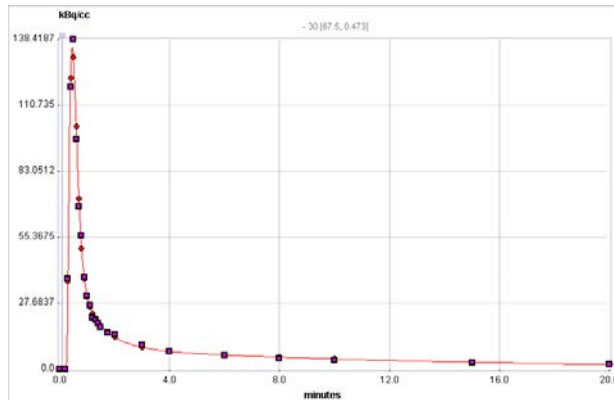
The blood and plasma activity concentrations require interpolation procedures for kinetic modeling, as described *above* (on page 51).

## Operational Model Curve

This **Gamma + 2 Exponentials** model is very similar to the *Compartment Model with 3 Eigenvalues* (on page 186). However, it includes a power term  $u^B$  which allows better

modeling of sharp peaks.

$$C_{\text{continuous}}(t) = \begin{cases} 0 & t < \text{Begin} \\ [A_1 u^B - A_2 - A_3] e^{-u \ln 2 / T_1} + A_2 e^{-u \ln 2 / T_2} + A_3 e^{-u \ln 2 / T_3} & t \geq \text{Begin}, u = t - \text{Begin} \end{cases}$$



## Parameter Fitting

The **Gamma + 2 Exponentials** model includes a **Begin** parameter, three **Amplitudes** ( $A_1$ ,  $A_2$ ,  $A_3$ ) and a **Halftimes** ( $T_1$ ,  $T_2$ ,  $T_3$ ), and a **Power Exponent** ( $B$ ). This functional form supports a peak rising from time **Begin** followed by an exponentially decreasing tail. The initial default of the **Power Exponent** is a fixed value of 1 which corresponds to the **Compartment Model with 3 Eigenvalues** (on page 186). Use the **Fit plasma activity** or **Fit whole blood** button to fit the operational model curve to the measurements. Then, for better fitting of the peak, enable the **Power Exponent** and fit again. The form of the curve reacts very sensitively to changes of the parameters. Note that all samples are considered, not only the ones after **Begin**.

The **Delay** parameter serves for correcting a timing offset between tissue and blood data. Positive delays correspond to delayed blood information and hence shift the blood curve to earlier times (to the left). The **Delay** is only relevant during the fitting of the tissue model where it can be *fitted as an additional parameter* (on page 77) to the parameters of the kinetic model.

## Simulation of Bolus/Infusion Tracer Application

When a compartment system has equilibrated, the total distribution volume can be calculated easily as the ratio of tracer concentration in tissue to the metabolite-corrected plasma concentration with a single static scan. It has been found that the time required to reach equilibrium can be shortened by an optimized tracer delivery. A setup which does not require a sophisticated tracer delivery system is to apply an initial bolus and continue with a constant infusion.

Carson et al. [1] have developed a method to optimize the ratio between the activity given as the initial bolus and the activity level of the subsequent infusion for quickly reaching an equilibrium (Appendix B in [1]). This method is *implemented* (on page 174) as a tissue model in PKIN. The current blood model **Bolus/Infusion** is only a visualization model and should not be used for fitting measured blood data. It allows entering the  $K_{\text{bol}}$  value found with the tissue model optimization and inspecting the plasma activity curve under the bolus/infusion regime.

## Operational Model Curve

For a bolus and infusion (B/I) experiment the expected input function can be derived from the input curve  $C_p(t)$  of a bolus-only experiment by

$$C_{\text{Model}}(t) = \frac{K_{\text{bol}} C_p(t) + \int_0^t C_p(\tau) d\tau}{K_{\text{bol}} + T}$$

$K_{\text{bol}}$  [min] defines the dose of the bolus expressed as minutes of infusion, and  $T$  is the total duration of the infusion (end of last frame).

## Parameter Fitting

This model cannot be used for data fitting. It assumes the plasma activity curve of a bolus experiment has been loaded and visualizes the expected input curve if **Kbol** [min] equivalents of the infusion are applied as an initial bolus. The criterion of fast equilibration is that the model curve gets soon constant and remains so.

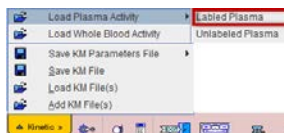
## Reference

1. Carson RE, Channing MA, Blasberg RG, Dunn BB, Cohen RM, Rice KC, Herscovitch P: Comparison of bolus and infusion methods for receptor quantitation: application to [18F]cyclofoxy and positron emission tomography. J Cereb Blood Flow Metab 1993, 13(1):24-42. DOI <http://dx.doi.org/10.1038/jcbfm.1993.6>

## Flumazenil, HOT Input Curve

The **Multiinjection, HOT** model is used in combination with the tissue model for multi-injection studies with  $^{11}\text{C}$ -Flumazenil as described by Delforge et al. [1]. Please refer to the documentation of the *implemented tissue model* (on page 125) for a more detailed description.

It is assumed that the total plasma activity of the tracer  $C_{\text{Plasma}}(t)$  is loaded from the menu as illustrated below.

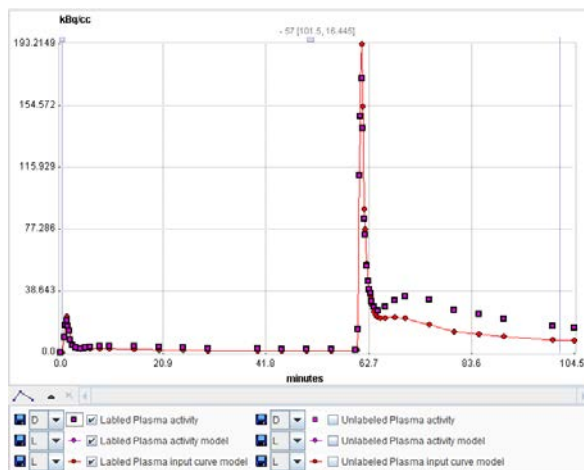




## Operational Model Curve

The input curve is then calculated by applying an exponential metabolite correction which considers the two injections of hot ligand at time **T1** and **T3**:

$$C_P(t) = \begin{cases} C_{Plasma} (A_1 e^{B_1(t-T_1)} + A_2 e^{B_2(t-T_1)}) & T_1 < t < T_3 \quad \text{decay since 1st injection} \\ C_{Plasma} (A_1 e^{B_1(t-T_3)} + A_2 e^{B_2(t-T_3)}) & t > T_3 \quad \text{decay since 3rd injection} \end{cases}$$



## Parameter Fitting

Note that although the corresponding parameters have a fit box, they should normally be used for entering pre-determined values of injection times and metabolite correction parameters. The default parameters of the for the bi-exponential metabolite correction (**A1**, **T1**, **A2**, **B2**) used in this model are from [1].

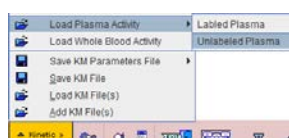
## Reference

Delforge J, Pappata S, Millet P, Samson Y, Bendriem B, Jobert A, Crouzel C, Syrota A: Quantification of benzodiazepine receptors in human brain using PET, [11C]flumazenil, and a single-experiment protocol. J Cereb Blood Flow Metab 1995, 15(2):284-300. DOI <http://www.nature.com/jcbfm/journal/v15/n2/pdf/jcbfm199534a.pdf>

## Flumazenil, COLD Input Curve

The **Multiinjection, COLD** model is used in combination with the tissue model for multi-injection studies with <sup>11</sup>C-Flumazenil as described by Delforge et al. [1]. Please refer to the documentation of the *implemented tissue model* (on page 125) for a more detailed description.

It is assumed that the total plasma activity of the labeled tracer  $C_{Plasma}(t)$  is loaded from the menu as illustrated below.



## Operational Model Curve

The plasma activity of unlabeled (cold) ligand is calculated by using the information of the labeled (hot) ligand, scaling it proportionally to the applied dose, and applying the same metabolite correction. The metabolite corrected plasma activity of the labeled compound  $C_p(t)$  is calculated by

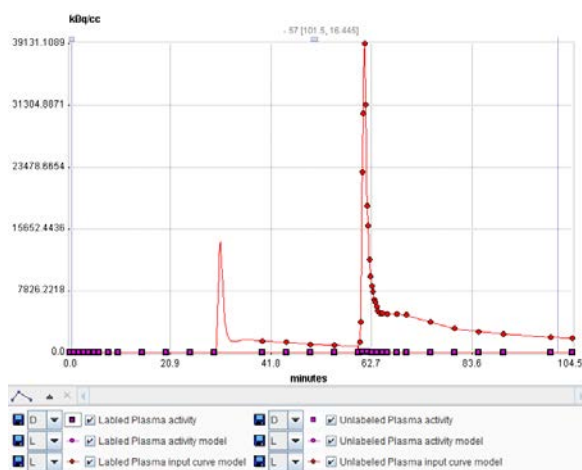
$$C_p(t) = \begin{cases} C_{Plasma} \left( A_1 e^{B_1(t-T_1)} + A_2 e^{B_2(t-T_1)} \right) & T_1 < t < T_3 \quad \text{decay since 1st injection} \\ C_{Plasma} \left( A_1 e^{B_1(t-T_3)} + A_2 e^{B_2(t-T_3)} \right) & t > T_3 \quad \text{decay since 3rd injection} \end{cases}$$

The calculation of the plasma activity of the unlabeled ligand  $C_u(t)$  is also performed differently in two time segments:

$$C_u(t) = \begin{cases} Dose_2 / Dose_1 C_p(t - T_2) & T_2 < t < T_3 \\ Dose_3^{Cold} / Dose_3^{Hot} C_p(t - T_3) + C_u^P(T_3) \left( A_1 e^{B_1(t-T_3)} + A_2 e^{B_2(t-T_3)} \right) & t > T_3 \end{cases}$$

- 1) Following the cold-only injection at  $T_2$  the input curve of the first hot injection at  $T_1$  is scaled according to the dose ration and shifted to start at  $T_2$ .

Following the injection at  $T_3$  with hot and cold ligand the input curve of the hot ligand is scaled, and a contribution of the first cold injection added which is decreasing by the metabolite build-up.



## Parameter Fitting

Note that although the corresponding parameters have a fit box, they should normally be used for entering pre-determined values of injection times and metabolite correction parameters. The default parameters of the for the bi-exponential metabolite correction (**A1**, **T1**, **A2**, **B2**) used in this model are from [1].

## Reference

Delforge J, Pappata S, Millet P, Samson Y, Bendriem B, Jobert A, Crouzel C, Syrota A: Quantification of benzodiazepine receptors in human brain using PET, [11C]flumazenil, and a single-experiment protocol. J Cereb Blood Flow Metab 1995, 15(2):284-300. DOI <http://www.nature.com/jcbfm/journal/v15/n2/pdf/jcbfm199534a.pdf>

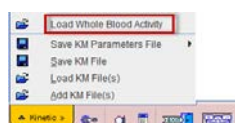
## Fraction Models for Plasma/Whole-Blood Activity

The following models for converting whole-blood activity into plasma activity are available in PKIN. Note that it is feasible to combine the conversion into plasma activity and then into authentic tracer activity into a single conversion function. In this case the parent fraction can remain the identity function, which is its default.

### Fix Plasma Fraction

The sequential steps for converting blood measurements to an input curve are explained in a *dedicated section* (on page 51). The current *plasma fraction* (on page 53) model serves for converting a whole-blood activity curve into a plasma-activity curve.

It is assumed that a whole-blood activity curve has been loaded with **Load Whole Blood Activity** entry in the menu.



With the **Fix** model the activity concentration in plasma is assumed to be proportional to the activity concentration in whole blood. The proportionality constant **Fraction** has to be specified by the user and has a default of 1.

Note that as long as no other plasma-related data is loaded or the plasma fraction changed, the **Fix** model is applied so that the whole-blood activity curve will be used as the input curve.

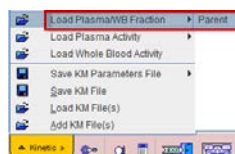
### Parameter Fitting

There is nothing to fit with the **Fix** model.

## Linear Interpolation of Plasma/Whole-blood Fraction

The sequential steps for converting blood measurements to an input curve are explained in a *dedicated section* (on page 51). The current *plasma fraction* (on page 53) model serves for converting a whole-blood activity curve into a plasma-activity curve.

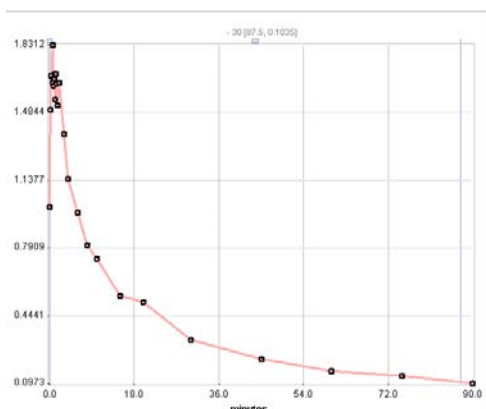
It is assumed that a curve representing the measured fractions of plasma/whole-blood activity has been loaded with **Load Plasma/WB Fraction**.



The **Lin. Interpolation** model interpolates linearly between the measured fraction values. Outside the measured values the following interpolation rules apply:

- ▶▶ The fraction at time  $t=0$  is assumed to equal 1.
- ▶▶ The fraction is assumed to remain constant after the last measured fraction.

For incomplete fraction data it is therefore recommended to replace the tail of the measurements by a fitted *tri-exponential* (on page 195) model.



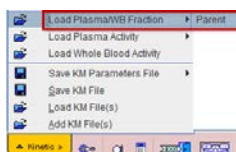
## Parameter Fitting

There is nothing to fit with the **Lin. Interpolation** model.

## Exponential Plasma/Whole-blood Fraction

The sequential steps for converting blood measurements to an input curve are explained in a *dedicated section* (on page 51). The current *plasma fraction* (on page 53) model serves for converting a whole-blood activity curve into a plasma-activity curve.

It is assumed that a curve representing the measured fractions of plasma/whole-blood activity has been loaded with **Load Plasma/WB Fraction**.

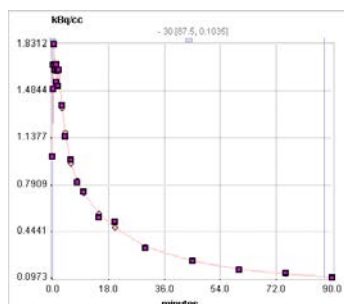


## Operational Model Curve

The **3 Exponentials** model allows replacing the tail of the fraction curve by a fitted sum of up to three exponentials. The **Begin** parameter defines at what time the models switches from linear to tri-exponential interpolation. The exponentials are each defined by an **Amplitude** and a **Halftime** [min] of the decay. The number of exponentials can easily be reduced by fixing one or two of the amplitudes at a value of 0.

$$f_{\text{Plasma/WB}}(t) = \begin{cases} \text{Lin.Interpol} & t < \text{Begin} \\ \sum_{i=1}^3 A_i e^{-u \ln 2 / T_i} & t \geq \text{Begin}, u = t - \text{Begin} \end{cases}$$

The illustration below shows an example with the exponentials fitted following 2 minutes (**Begin** = 120sec). Note that the linear interpolation assumes a fraction of 1 at t=0.



### Parameter Fitting

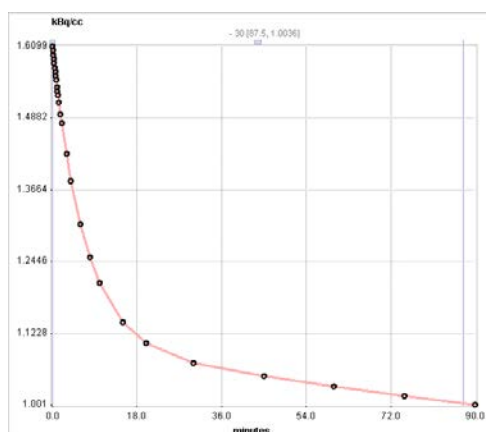
The model supports the fitting of the parameters **Begin**, **Amplitude 1, 2, 3** ( $=A_1, A_2, A_3$ ) and **Halftime 1, 2, 3** ( $=T_1, T_2, T_3$ ). Fitting is started with the **Fit plasma fraction** button, which only takes the samples after the **Begin** time into consideration.

Appropriate default parameters for FDG scans:

- ▶▶ Rat experiments [1]: Plasma/whole\_blood(t) =  $0.51 e^{(-\ln 2/4.79 t)} + 0.3 e^{(-\ln 2/337 t)} + 0.8$
- ▶▶ Mouse Experiments [2]: Plasma/whole\_blood(t) =  $0.386 e^{(-0.191 t)} + 1.165$

### Use without Measured Fraction

If no measured plasma/whole-blood fraction data is available, the **3 Exponentials** model can still be applied. In this case the **Begin** time is disregarded, and the functional defined by the entered parameters applied.



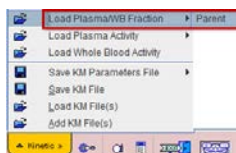
### References

- 1) Weber B, Burger C, Biro P, Buck A: A femoral arteriovenous shunt facilitates arterial whole blood sampling in animals. Eur J Nucl Med Mol Imaging 2002, 29(3):319-323.
- Ferl GZ, Zhang X, Wu HM, Kreissl MC, Huang SC: Estimation of the  $^{18}\text{F}$ -FDG input function in mice by use of dynamic small-animal PET and minimal blood sample data. J Nucl Med 2007, 48(12):2037-2045. DOI <http://dx.doi.org/10.2967/jnumed.107.041061>

## Hill Function

The sequential steps for converting blood measurements to an input curve are explained in a *dedicated section* (on page 51). The current *plasma fraction* (on page 53) model serves for converting a whole-blood activity curve into a plasma-activity curve.

It is assumed that a curve representing the measured fractions of plasma/whole-blood activity has been loaded with **Load Plasma/WB Fraction**.

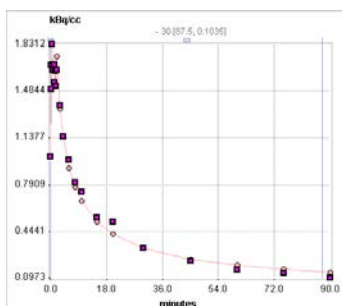


## Operational Model Curve

The **Hill** model allows replacing the tail of the fraction curve by an analytical function. The **Begin** parameter defines at what time the models switches from linear interpolation to the Hill function.

$$f_{\text{Plasma/WB}}(t) = \begin{cases} \text{Lin. Interpol} & t \leq \text{Begin} \\ \text{scale} \cdot \left[ 1 - \left( \frac{A u^B}{u^B + C} \right) \right] & t > \text{Begin}, u = t - \text{Begin} \end{cases}$$

The illustration below shows an example with the Hill function fitted following 2 minutes (**Begin** = 120sec). The the weighting has been changed for putting more emphasis on the tail. Note that the linear interpolation assumes a fraction of 1 at t=0.



## Parameter Fitting

The model supports the fitting of the parameters **Begin**, **scale**, **A**, **B**, **C**. Fitting is started with the **Fit plasma fraction** button, which only takes the samples after the **Begin** time into consideration.

## Use without Measured Fraction

If no measured plasma/whole-blood fraction data is available, the **Hill** model can still be applied. In this case the **Begin** time is disregarded, and the functional defined by the entered parameters applied.

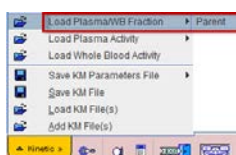
## References

1. Gunn RN, Sargent PA, Bench CJ, Rabiner EA, Osman S, Pike VW, Hume SP, Grasby PM, Lammertsma AA: Tracer kinetic modeling of the 5-HT1A receptor ligand [carbonyl-11C]WAY-100635 for PET. *Neuroimage* 1998, 8(4):426-440. *DOI* <http://dx.doi.org/10.1006/nimg.1998.0379>

## Watabe Function

The sequential steps for converting blood measurements to an input curve are explained in a *dedicated section* (on page 51). The current *plasma fraction* (on page 53) model serves for converting a whole-blood activity curve into a plasma-activity curve.

It is assumed that a curve representing the measured fractions of plasma/whole-blood activity has been loaded with **Load Plasma/WB Fraction**.

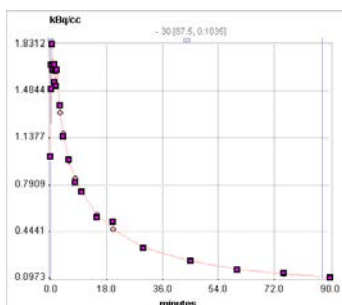


## Operational Model Curve

The **Watabe** model allows replacing the tail of the fraction curve by an analytical function. The **Begin** parameter defines at what time the models switches from linear interpolation to the Hill function.

$$f_{\text{Plasma/WB}}(t) = \begin{cases} \text{Lin. Interpol} & t \leq \text{Begin} \\ \frac{\text{scale}}{[1 + (Au)^B]^C} & t > \text{Begin}, u = t - \text{Begin} \end{cases}$$

The illustration below shows an example with the Hill function fitted following 2 minutes (**Begin** = 120sec). The the weighting has been changed for putting more emphasis on the tail. Note that the linear interpolation assumes a fraction of 1 at  $t=0$ .



## Parameter Fitting

The model supports the fitting of the parameters **Begin**, **scale**, **A**, **B**, **C**. Fitting is started with the **Fit plasma fraction** button, which only takes the samples after the **Begin** time into consideration.

## Use without Measured Fraction

If no measured plasma/whole-blood fraction data is available, the **Watabe** model can still be applied. In this case the **Begin** time is disregarded, and the functional defined by the entered parameters applied.

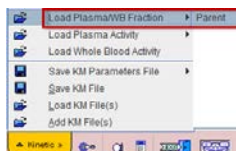
## References

1. Watabe H, Channing MA, Der MG, Adams HR, Jagoda E, Herscovitch P, Eckelman WC, Carson RE: Kinetic analysis of the 5-HT<sub>2A</sub> ligand [11C]MDL 100,907. J Cereb Blood Flow Metab 2000, 20(6):899-909. DOI <http://dx.doi.org/10.1097/00004647-200006000-00002>

## Sigmoid Function

The sequential steps for converting blood measurements to an input curve are explained in a *dedicated section* (on page 51). The current *plasma fraction* (on page 53) model serves for converting a whole-blood activity curve into a plasma-activity curve.

It is assumed that a curve representing the measured fractions of plasma/whole-blood activity has been loaded with **Load Plasma/WB Fraction**.

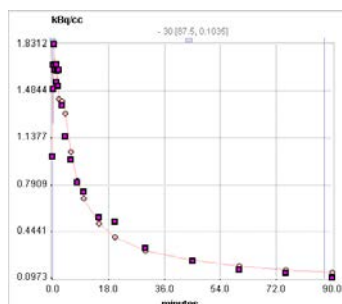


## Operational Model Curve

The **Sigmoid** model allows replacing the tail of the fraction curve by an analytical function. The **Begin** parameter defines at what time the models switches from linear interpolation to the Hill function.

$$f_{Plasma/WB}(t) = \begin{cases} Lin.Interpol & t \leq Begin \\ scale \cdot \left[ \left( 1 - \frac{u^3}{u^3 + 10^A} \right)^B + C \right] / (1 + C) & t > Begin, u = t - Begin \end{cases}$$

The illustration below shows an example with the Hill function fitted following 2 minutes (**Begin** = 120sec). The the weighting has been changed for putting more emphasis on the tail. Note that the linear interpolation assumes a fraction of 1 at t=0.





## Parameter Fitting

The model supports the fitting of the parameters **Begin**, **scale**, **A**, **B**, **C**. Fitting is started with the **Fit plasma fraction** button, which only takes the samples after the **Begin** time into consideration.

## Use without Measured Fraction

If no measured plasma/whole-blood fraction data is available, the **Sigmoid** model can still be applied. In this case the **Begin** time is disregarded, and the functional defined by the entered parameters applied.

## References

1. Owen DR, Guo Q, Kalk NJ, Colasanti A, Kalogiannopoulou D, Dimber R, Lewis YL, Libri V, Barletta J, Ramada-Magalhaes J, Kamalakaran A, Nutt DJ, Passchier J, Matthews PM, Gunn RN, Rabiner EA. Determination of [(11)C]PBR28 binding potential in vivo: a first human TSPO blocking study. *J Cereb Blood Flow Metab.* 2014;34(6):989-94. *DOI* <http://dx.doi.org/10.1038/jcbfm.2014.46>

## Fraction Models for Parent/Plasma Activity

The following parent fraction models are available in PKIN.

### Fix Parent/Plasma Fraction

The sequential steps for converting blood measurements to an input curve are explained in a *dedicated section* (on page 51). The current *parent fraction* (on page 54) model serves for converting a plasma activity curve into an input curve (authentic, unchanged tracer in plasma).

#### Operational Model Curve

The **Fix** model assumes that the concentration of authentic tracer in plasma is in constant proportion to the concentration of total tracer in plasma. The fraction  $f_{\text{parent}}(t)$  of authentic tracer to total tracer in plasma is therefore given by

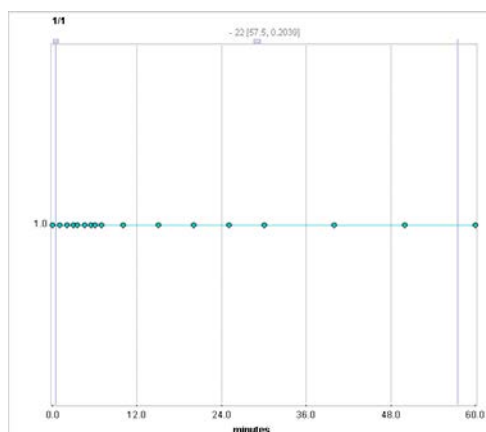
$$f_{\text{parent}}(t) = f_p$$

with  $f_p$  representing the free fraction of authentic tracer in plasma, i.e. the fraction of tracer not bound to plasma proteins.

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**Note:** The **Fix** model is the default as soon as blood data is loaded. This is adequate if a metabolite correction is not necessary or has been performed in a prior step, for instance externally before loading the plasma activity.

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#### Parameter Fitting

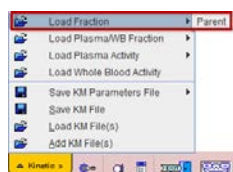
There is nothing to fit with the **Lin. Interpolation** model. The only input parameter is the free fraction **fp** which defaults to 1, as it is experimentally difficult to measure.

### Linear Interpolation Fraction

The sequential steps for converting blood measurements to an input curve are explained in a *dedicated section* (on page 51). The current *parent fraction* (on page 54) model serves for

converting a plasma activity curve into an input curve (authentic, unchanged tracer in plasma).

The **Lin. Interpolation** model assumes that the concentration ratio of authentic tracer to total tracer in plasma has been determined at certain times during the acquisition and loaded with the **Load Fraction** entry in the menu.



## Operational Model Curve

The **Lin. Interpolation** model interpolates linearly between the measured fraction values  $F_k$ .

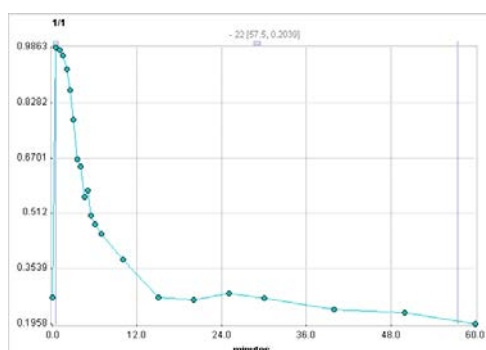
$$f_{parent}(t) = f_p \cdot Interpol(F_k)$$

and multiplies with  $f_p$  representing the free fraction of authentic tracer in plasma, i.e. the fraction of tracer not bound to plasma proteins.

Outside the measured values the following interpolation rules apply:

- ▶▶ The fraction at time  $t=0$  is assumed to equal 1.
- ▶▶ The fraction is assumed to remain constant after the last measured fraction. For incomplete fraction data it is therefore recommended to replace the tail of the measurements by a fitted *tri-exponential* (on page 205) model.

Note that fraction values  $>1$  are not allowed.



## Parameter Fitting

There is nothing to fit with the **Lin. Interpolation** model. The only input parameter is the free fraction **fp** which defaults to 1, as it is experimentally difficult to measure.

## 1-Exponential

The sequential steps for converting blood measurements to an input curve are explained in a *dedicated section* (on page 51). The current *parent fraction* (on page 54) model serves for

converting a plasma activity curve into an input curve (authentic, unchanged tracer in plasma).

The **1-Exponential** model assumes that the concentration ratio of authentic tracer to total tracer in plasma has been determined at certain times during the acquisition and loaded with the **Load Fraction** entry in the menu.



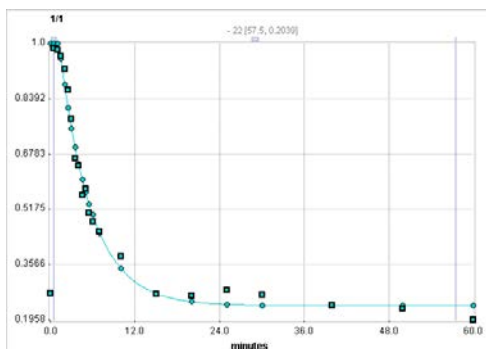
## Operational Model Curve

The **1-Exponential** model applied by Wu et al [1] for modeling of the parent fraction has the following functional form

$$f_{parent}(t) = f_p \cdot \begin{cases} 1 & t \leq \text{Begin} \\ Ae^{-u \ln 2 / T} + (1 - A) & t > \text{Begin}, u = t - \text{Begin} \end{cases}$$

The calculated fraction is multiplied with  $f_p$ , the free fraction of authentic tracer in plasma, i.e. the fraction of tracer not bound to plasma proteins.

The fraction calculation assumes negligible metabolites before a **Begin** time, and therefore remains constant at 1. From **Begin** on it decreases exponentially towards an equilibration fraction (1-A) with halftime T.



## Parameter Fitting

The free fraction **fp** is an input parameter which has no impact on fitting the model curve to the fraction data. The default for **fp** is 1 as it is experimentally difficult to measure.

The model supports the fitting of the parameters **Begin**, **Scale** (=A) and **Halftime**. All data samples are considered in the fitting process irrespective of **Begin**. The **Scale** parameter is restricted to the range [0,1] per default.

## Use without Measured Parent Fraction

The **1-Exponential** parent fraction model can be applied even if no parent fraction measurements were loaded. In this case the user has to specify a parameter set which establishes a representative metabolite correction for the used tracer.

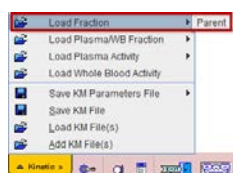
## Reference

1. Wu S, Ogden RT, Mann JJ, Parsey RV: Optimal metabolite curve fitting for kinetic modeling of 11C-WAY-100635. J Nucl Med 2007, 48(6):926-931. DOI 10.2967/jnumed.106.038075

## 2-Exponentials

The sequential steps for converting blood measurements to an input curve are explained in a *dedicated section* (on page 51). The current *parent fraction* (on page 54) model serves for converting a plasma activity curve into an input curve (authentic, unchanged tracer in plasma).

The **2-Exponential** model assumes that the concentration ratio of authentic tracer to total tracer in plasma has been determined at certain times during the acquisition and loaded with the **Load Fraction** entry in the menu.



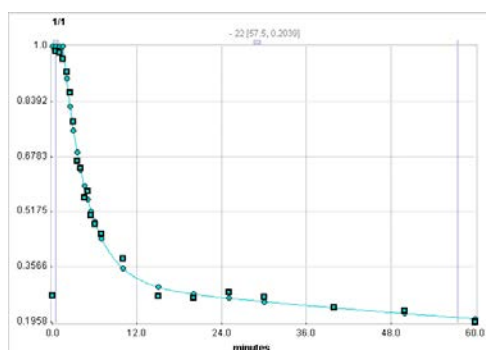
## Operational Model Curve

The **2-Exponential** model applied by Wu et al [1] for modeling of the parent fraction has the following functional form

$$f_{parent}(t) = f_p \cdot \begin{cases} 1 & t \leq \text{Begin} \\ A e^{-u \ln 2 / T_1} + (1 - A) e^{-u \ln 2 / T_2} & t > \text{Begin}, u = t - \text{Begin} \end{cases}$$

The calculated fraction is multiplied with  $f_p$ , the free fraction of authentic tracer in plasma, i.e. the fraction of tracer not bound to plasma proteins.

The fraction calculation assumes negligible metabolites before a **Begin** time, and therefore remains constant at 1. From **Begin** on it decreases as a sum of two exponentials with halftimes  $T_1$  and  $T_2$ . Note that because only one scale factor  $A$  is used, continuity at time **Begin** is ensured.



## Parameter Fitting

The free fraction **fp** is an input parameter which has no impact on fitting the model curve to the fraction data. The default for **fp** is 1 as it is experimentally difficult to measure.

The model supports the fitting of the parameters **Begin**, **Scale (=A)**, **Halftime 1** and **Halftime 2**. All data samples are considered in the fitting process irrespective of **Begin**. The **Scale** parameter is restricted to the range [0,1] per default.

## Use without Measured Parent Fraction

The **2-Exponential** parent fraction model can be applied even if no parent fraction measurements were loaded. In this case the user has to specify a parameter set which establishes a representative metabolite correction for the used tracer.

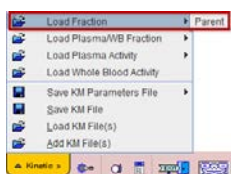
## Reference

1. Wu S, Ogden RT, Mann JJ, Parsey RV: Optimal metabolite curve fitting for kinetic modeling of 11C-WAY-100635. J Nucl Med 2007, 48(6):926-931. DOI 10.2967/jnumed.106.038075

## 3-Exponentials

The sequential steps for converting blood measurements to an input curve are explained in a *dedicated section* (on page 51). The current *parent fraction* (on page 54) model serves for converting a plasma activity curve into an input curve (authentic, unchanged tracer in plasma).

The **3-Exponential** model assumes that the concentration ratio of authentic tracer to total tracer in plasma has been determined at certain times during the acquisition and loaded with the **Load Fraction** entry in the menu.



## Operational Model Curve

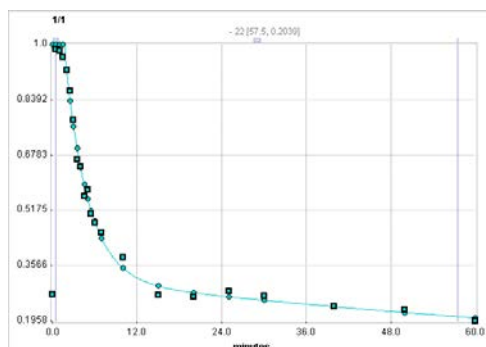
The general **3-Exponential** model for the parent fraction has the following functional form

$$f_{parent}(t) = f_p \cdot \begin{cases} 1 & t \leq \text{Begin} \\ A_1 e^{-u \ln 2 / T_1} + A_2 e^{-u \ln 2 / T_2} + A_3 e^{-u \ln 2 / T_3} & t > \text{Begin}, u = t - \text{Begin} \end{cases}$$

The calculated fraction is multiplied with  $f_p$ , the free fraction of authentic tracer in plasma, i.e. the fraction of tracer not bound to plasma proteins.

The fraction calculation assumes negligible metabolites before a **Begin** time, and therefore remains constant at 1. From **Begin** on it decreases as a sum of three exponentials with

halftimes  $T_1$ ,  $T_2$  and  $T_3$ . Note that the calculated fractions are restricted to the physiologic [0,1] value range.



## Parameter Fitting

The free fraction **fp** is an input parameter which has no impact on fitting the model curve to the fraction data. The default for **fp** is 1 as it is experimentally difficult to measure.

The model supports the fitting of the parameters **Begin**, **Amplitude 1, 2, 3** ( $=A_1, A_2, A_3$ ) and **Halftime 1, 2, 3** ( $=T_1, T_2, T_3$ ). All data samples are considered in the fitting process irrespective of **Begin**. The three **Amplitude** parameters are restricted to the range [0,1] per default.

## Use without Measured Parent Fraction

The **3-Exponential** parent fraction model can be applied even if no parent fraction measurements were loaded. In this case the user has to specify a parameter set which establishes a representative metabolite correction for the used tracer.

## Hill Function

The sequential steps for converting blood measurements to an input curve are explained in a *dedicated section* (on page 51). The current *parent fraction* (on page 54) model serves for converting a plasma activity curve into an input curve (authentic, unchanged tracer in plasma).

The **Hill** model assumes that the concentration ratio of authentic tracer to total tracer in plasma has been determined at certain times during the acquisition and loaded with the **Load Fraction** entry in the menu.



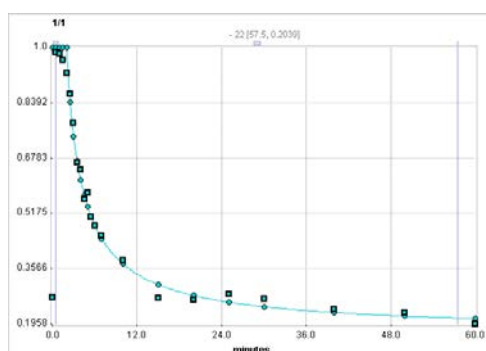
## Operational Model Curve

The **Hill** model applied by Gunn et al [1] for modeling of the parent fraction has the following functional form

$$f_{parent}(t) = f_p \cdot \begin{cases} 1 & t \leq \text{Begin} \\ 1 - \left( \frac{A u^B}{u^B + C} \right) & t > \text{Begin}, u = t - \text{Begin} \end{cases}$$

whereby the calculated fraction is multiplied with  $f_p$ , the free fraction of authentic tracer in plasma, i.e. the fraction of tracer not bound to plasma proteins.

The fraction calculation assumes negligible metabolites before a **Begin** time, and therefore remains constant at 1. From **Begin** on it decreases smoothly.



## Parameter Fitting

The free fraction **fp** is an input parameter which has no impact on fitting the **Hill** curve to the fraction data. The default for **fp** is 1 as it is experimentally difficult to measure.

The model supports the fitting of the parameters **Begin**, **A**, **B** and **C**. All data samples are considered in the fitting process irrespective of **Begin**. The scaling parameter **A** is restricted to the range [0,1] per default.

## Use without Measured Parent Fraction

The **Hill** parent fraction model can be applied even if no parent fraction measurements were loaded. In this case the user has to specify a parameter set which establishes a representative metabolite correction for the used tracer.

## Reference

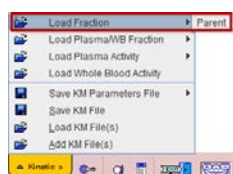
1. Gunn RN, Sargent PA, Bench CJ, Rabiner EA, Osman S, Pike VW, Hume SP, Grasby PM, Lammertsma AA: Tracer kinetic modeling of the 5-HT1A receptor ligand [carbonyl-11C]WAY-100635 for PET. *Neuroimage* 1998, 8(4):426-440. *DOI* <http://dx.doi.org/10.1006/nimg.1998.0379>



## Watabe Function

The sequential steps for converting blood measurements to an input curve are explained in a *dedicated section* (on page 51). The current *parent fraction* (on page 54) model serves for converting a plasma activity curve into an input curve (authentic, unchanged tracer in plasma).

The **Watabe** model assumes that the concentration ratio of authentic tracer to total tracer in plasma has been determined at certain times during the acquisition and loaded with the **Load Fraction** entry in the menu.



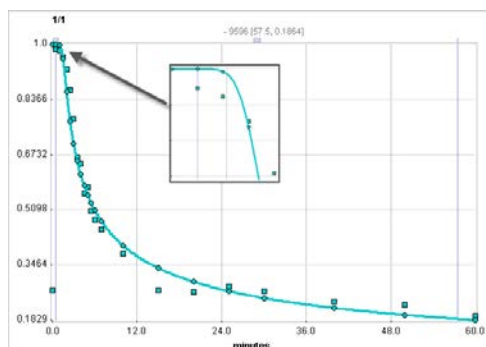
## Operational Model Curve

The **Watabe** model applied by Watabe et al [1] for modeling of the parent fraction has the following functional form

$$f_{parent}(t) = f_p \cdot \begin{cases} 1 & t \leq \text{Begin} \\ \frac{1}{[1 + \{Au\}^B]^C} & t > \text{Begin}, u = t - \text{Begin} \end{cases}$$

with **B** set to 2. The calculated fraction is multiplied with  $f_p$ , the free fraction of authentic tracer in plasma, i.e. the fraction of tracer not bound to plasma proteins.

The fraction calculation assumes negligible metabolites before a **Begin** time, and therefore remains constant at 1. From **Begin** on it decreases smoothly.



## Parameter Fitting

The free fraction **fp** is an input parameter which has no impact on fitting the model curve to the fraction data. The default for **fp** is 1 as it is experimentally difficult to measure.

The model supports the fitting of the parameters **Begin**, **A**, **B** and **C**. All data samples are considered in the fitting process irrespective of **Begin**. The scaling parameter **A** is restricted to the range [0,1] per default.

## Use without Measured Parent Fraction

The **Watabe** parent fraction model can be applied even if no parent fraction measurements were loaded. In this case the user has to specify a parameter set which establishes a representative metabolite correction for the used tracer.

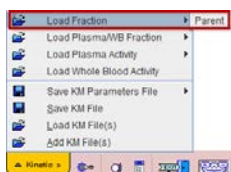
## Reference

1. Watabe H, Channing MA, Der MG, Adams HR, Jagoda E, Herscovitch P, Eckelman WC, Carson RE: Kinetic analysis of the 5-HT<sub>2A</sub> ligand [11C]MDL 100,907. J Cereb Blood Flow Metab 2000, 20(6):899-909. DOI <http://dx.doi.org/10.1097/00004647-200006000-00002>

## Power-Damped Exponentials

The sequential steps for converting blood measurements to an input curve are explained in a *dedicated section* (on page 51). The current *parent fraction* (on page 54) model serves for converting a plasma activity curve into an input curve (authentic, unchanged tracer in plasma).

The **Power-Damped Exp** model assumes that the concentration ratio of authentic tracer to total tracer in plasma has been determined at certain times during the acquisition and loaded with the **Load Fraction** entry in the menu.

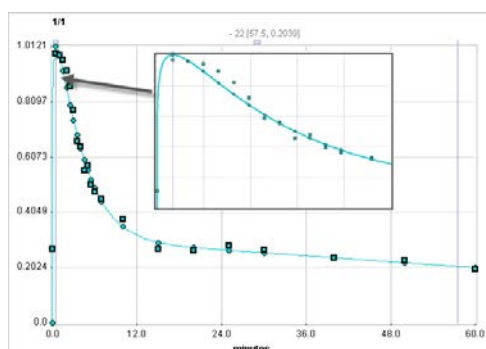


## Operational Model Curve

The general **Power-Damped Exp** model for the parent fraction handles the situation where the parent fraction starts at a low value and then increases, before dropping. It has the following functional form

$$f_{\text{parent}}(t) = f_p \cdot \begin{cases} 0 & t \leq 0 \\ t^{\alpha} (A_1 e^{-t \ln 2 / T_1} + A_2 e^{-t \ln 2 / T_2} + A_3 e^{-t \ln 2 / T_3}) & t > 0 \end{cases}$$

The calculated fraction is multiplied with  $f_p$ , the free fraction of authentic tracer in plasma, i.e. the fraction of tracer not bound to plasma proteins.



## Parameter Fitting

The free fraction **fp** is an input parameter which has no impact on fitting the model curve to the fraction data. The default for **fp** is 1 as it is experimentally difficult to measure.

The model supports the fitting of the parameters **alpha**, **Amplitude 1, 2, 3** ( $=A_1, A_2, A_3$ ) and **Halftime 1, 2, 3** ( $=T_1, T_2, T_3$ ). The three **Amplitude** parameters are restricted to the range [0,1] per default. However, the calculated fractions are *not* restricted to a certain value range.

## Use without Measured Parent Fraction

The **Power-Damped Exp** parent fraction model can be applied even if no parent fraction measurements were loaded. In this case the user has to specify a parameter set which establishes a representative metabolite correction for the used tracer.

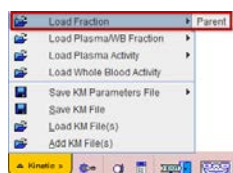
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<http://jnm.snmjournals.org/content/47/11/1796.full.pdf>

## Sigmoid Function

The sequential steps for converting blood measurements to an input curve are explained in a *dedicated section* (on page 51). The current *parent fraction* (on page 54) model serves for converting a plasma activity curve into an input curve (authentic, unchanged tracer in plasma).

The **Sigmoid** model assumes that the concentration ratio of authentic tracer to total tracer in plasma has been determined at certain times during the acquisition and loaded with the **Load Fraction** entry in the menu.

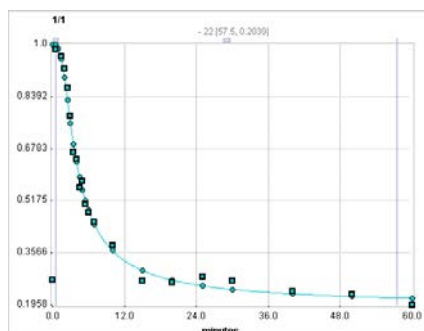


## Operational Model Curve

The general **Sigmoid** function for the parent fraction has been applied for [11C]PBR28 and has the following functional form [1]

$$f_{parent}(t) = f_p \cdot \left\{ \begin{array}{ll} 1 & t \leq \text{Begin} \\ \left( \left( 1 - \frac{u^3}{u^3 + 10^4} \right)^B + C \right) / (1 + C) & t > \text{Begin}, u = t - \text{Begin} \end{array} \right\}$$

The calculated fraction is multiplied with  $f_p$ , the free fraction of authentic tracer in plasma, i.e. the fraction of tracer not bound to plasma proteins.



### Parameter Fitting

The free fraction **fp** is an input parameter which has no impact on fitting the model curve to the fraction data. The default for **fp** is 1 as it is experimentally difficult to measure. The model supports the fitting of the parameters **A**, **B** and **C**.

### Use without Measured Parent Fraction

The **Sigmoid** parent fraction model can be applied even if no parent fraction measurements were loaded. In this case the user has to specify a parameter set which establishes a representative metabolite correction for the used tracer.

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**PMOD Technologies Ltd**

Sumatrastrasse 25  
8006 Zürich  
Switzerland  
+41 (44) 350 46 00  
support@pmod.com  
<http://www.pmod.com>

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